SIMULATION MODEL FOR CALCULATING THE PROBABILISTIC AND TEMPORAL CHARACTERISTICS OF THE PROJECT AND THE RISKS OF ITS UNTIMELY COMPLETION

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Abstract: The object of the research is a project containing a set of mutually dependent jobs with a random duration of execution. The goal is to build a simulation model that provides the ability to estimate a random variable describing the duration of the project, the distribution law of this quantity, as well as the probability of completing the project in a given time interval. The urgency of this task is due to the fact that existing approaches do not always differ with acceptable accuracy.

Key words: mutual dependence of jobs, random duration of service, probabilistic-temporal characteristics, PERT.

1. INTRODUCTION

Features of the functioning of modern complex service or production systems are strict requirements for the quality of the process of planning a project that is a set of interdependent activities. In particular, if the actual schedule differs from the planned one, this may lead to adverse consequences, including penalties, deterioration of the company's reputation, and others. The task is complicated by the presence of mutual dependence between individual jobs, the duration of which is a random variable. In this regard, a mathematical and software apparatus is needed that allows one to obtain the probabilistic-temporal characteristics of the project, which are distinguished by increased accuracy in comparison with existing analogues.

Currently, there are different approaches to solving this problem [1-9]. Among them, one can single out the PERT (Program Evaluation and Review Technique) method, which allows solving such problems as evaluating the distribution law up to parameters of a random variable describing the duration of the project and its numerical characteristics such as mathematical expectation and variance. However, studies have shown that not in all cases the prerequisites on which this method is based are true in practice. And, consequently, estimates of probabilistic-temporal characteristics will be inaccurate. Because of this, it becomes necessary to develop our own tools for obtaining assessment data, which would be distinguished by increased accuracy compared to analogs. Analysis of possible approaches has shown that an effective means of obtaining such estimates is the development of a simulation model, the input of which is the known time characteristics of individual jobs and their mutual dependence. The output will be an estimate of the distribution law of the duration of the project and its numerical characteristics.

2. ANALYSIS OF THE PERT METHOD FOR OBTAINING THE PROBABILISTIC-TIME CHARACTERISTICS OF THE PROJECT

Currently, the PERT method is usually used to assess the probabilistic and temporal characteristics of projects. This method works on the following assumptions:

- the duration of each job has a beta distribution;
- the smallest (a), the largest (b) and the most probable (m - mode) execution time of each job are known.

For each of jobs, the method made it possible to find estimates of the mathematical expectation and variance. They were offered the following formulas:

$$M\xi = \frac{a + 4m + b}{6}; \tag{1}$$

$$D\xi = \frac{(b - a)^2}{36} \tag{2}$$

Computational experiments designed to evaluate these numerical characteristics, described in [10], have shown that these estimates are, on the whole, fairly accurate.

In addition, the PERT method offers estimates of a random variable describing the duration of the entire project. The numerical characteristics of this value are extremely important, because, based on this value, the critical path and start time of each job are calculated. PERT estimates the duration of the entire project by a normal law with parameters:

$$M\eta = M\xi_1 + M\xi_2 + ... + M\xi_k, \tag{3}$$

$$\sigma\eta = \sqrt{D\xi_1 + D\xi_2 + ... + MD}. \tag{4}$$

Here $\xi_1, \ldots, \xi_k$ are random variables describing the duration of jobs on the critical path.

This hypothesis is based on the central limit theorem, which states that the distribution of a sufficiently large number of identically distributed terms can be approximated by a normal law.

However, in practice, even large projects may not contain many critical paths. In addition, the random variables describing the duration of individual jobs, despite the fact that they have a single assumed beta law, can vary within completely different limits, which contradicts the hypothesis that their distribution is identical. In this regard, the question arises about the accuracy of the PERT method estimates for any, including small projects with different durations of individual job.

3. A SIMULATION MODEL FOR ANALYZING THE PROBABILISTIC-TEMPORAL CHARACTERISTICS OF THE PROJECT

Let's conduct a computational experiment based on simulation modeling, which will allow estimating, first of all, the mathematical expectation of the duration of the project, since it is this value that is the basis in scheduling the implementation of individual jobs. In addition, it is advisable to determine the remaining characteristics of the investigated random variable and evaluate the law of its distribution.

As the initial data for modeling, it is necessary to submit a project specified by a set of mutually dependent works. Graphically, such a project can be presented as follows (Figure 1).

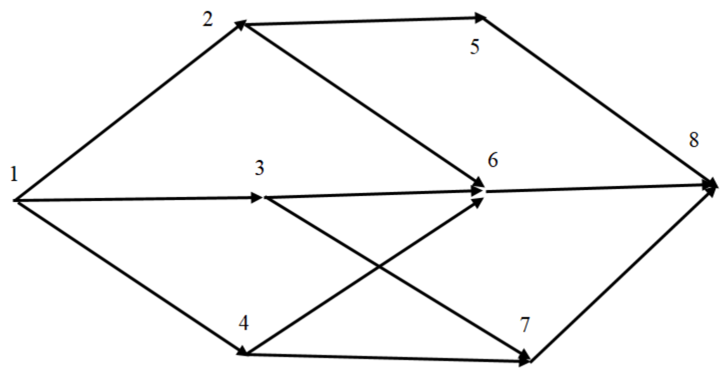

Fig. 1. General view of the project

Here, numbers indicate events that symbolize the beginning and the ending of jobs. In this case, it is more convenient to describe their mutual dependence. Each individual job must be specified by the duration in the form that is used in PERT: minimum and maximum execution time of jobs. In addition, since formula (1), as research has shown, is quite accurate, we will proceed from the assumptions that the mathematical expectation of the duration of each job is known. In this case, it is easy to show that the parameters p and q of the beta distribution, which describes the duration of each job, will be determined by the formulas [10]:

$$p = \frac{(b - M\xi) \cdot (M\xi - a)^2 - D\xi \cdot (M\xi - a)}{D\xi \cdot (b - a)} \qquad (5)$$

$$q = \frac{(b - M\xi)^2 \cdot (M\xi - a) - D\xi \cdot (b - M\xi)}{D\xi \cdot (b - a)}. \qquad (6)$$

The model should simulate the process of completing of jobs with a given dependence and duration, and at the output, as a result of multiple runs, obtain the following results:

- expected value (mathematical expectation);
- variance;
- estimate of the distribution density.

The implementation of the model will depend on the choice of the simulation environment. Unfortunately, at present there are no specialized applications focused on the ability to efficiently simulate the process of servicing a set of serial-parallel jobs defined by a graph, a possible example of which is shown in Figure 1. In this regard, the analysis of general-purpose simulation modelling environments was carried out, among which the AnyLogic environment can be distinguished [11]. This is a modern software product with a powerful graphical interface, as well as a large number of various libraries that greatly simplify the process of building and using models. In particular, the following advantages of AnyLogic are key for this task:

- the possibility of specifying the mutual dependence of the work in a form quite close to Fig. one;
- the ability to work with any distributions, in particular, the ability to set the duration of individual jobs by the beta law;
- a powerful statistical apparatus that allows the collection and statistical evaluation of the necessary characteristics and distribution laws.

Let's consider the specifics of the formation of a simulation model. Highlight in graph describing the interdependence of jobs is the ability to start the job only after the completion of all related jobs and the ability to parallelize the jobs. In the first case, it's necessary to specify an object that allows going to the next step of the model only if the previous events are completed (in this case, the previous work). The AnyLogic environment has an Assembler element that can effectively perform this task (Figure 2).

In Figure 2 the object «Event7», modeled by the Assembler, will prevent the work modeled with the pair of queue12 and job12 objects from starting (possibly waiting if the work cannot be done at the moment and simulating the job) until the job7 and the job8 are done.

In order to parallelize two or more jobs, there is a SPLIT object for copying incoming objects. Figure 3 shows the parallelization of two jobs.

If it's necessary to parallelize more jobs, you will need several SPLIT objects.

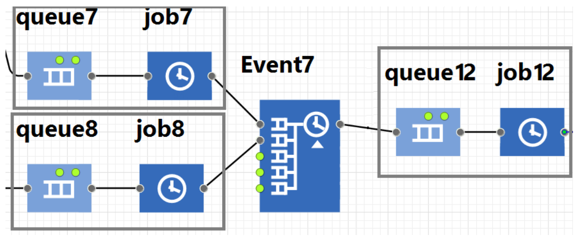

Fig. 2. Illustration of heuristic 2

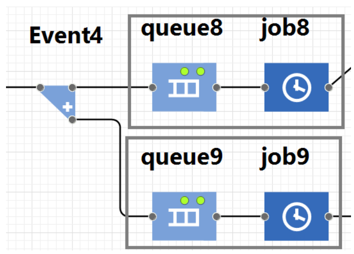

Fig. 3. Parallelization of jobs

The next step is to determine the duration of the job. The algorithm will be as follows:

- set the parameters a and b for each job, as well as its mathematical expectation;
- estimate the variance according to the (2);
- find the parameters p and q of the beta distribution, which will simulate the service duration, using (5) and (6). The dependence between the parameters a, b and m (in this case, the mathematical expectation) and the variables describing the variance and the values of *p* and *q* can be easily performed using the elements of the AnyLogic environment system dynamics library. For each of job, its own dependence is built, the general view of which is shown in Figure 4.

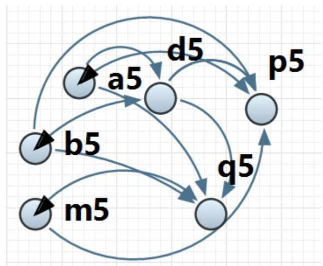

Fig. 4. Dependency between parameters and variables

Thus, the main mechanisms are described that allow obtaining a simulation model for assessing the probabilistic-temporal characteristics of the project.

In particular, for a project defined using the graph shown in Figure 1, the following model was obtained (Figure 5).

Let's give an example of how this model works. To do this, pre-assign all the numerical values to the works using the parameters shown in Figure 4. Let's consider a special case when all jobs are given by equally asymmetric random variables with parameters: a = 3; b = 7; m = 6.

Let's calculate the duration of the project using the PERT method. To do this, let us single out the critical path for it. In this case, any path from vertex 1 to vertex 8 will be critical, since the duration is set identically. In particular, the project under study will have 6 critical paths:

1-2-5-8; 1-2-6-8; 1-3-6-8; 1-3-7-8; 1-4-6-8; 1-4-7-8

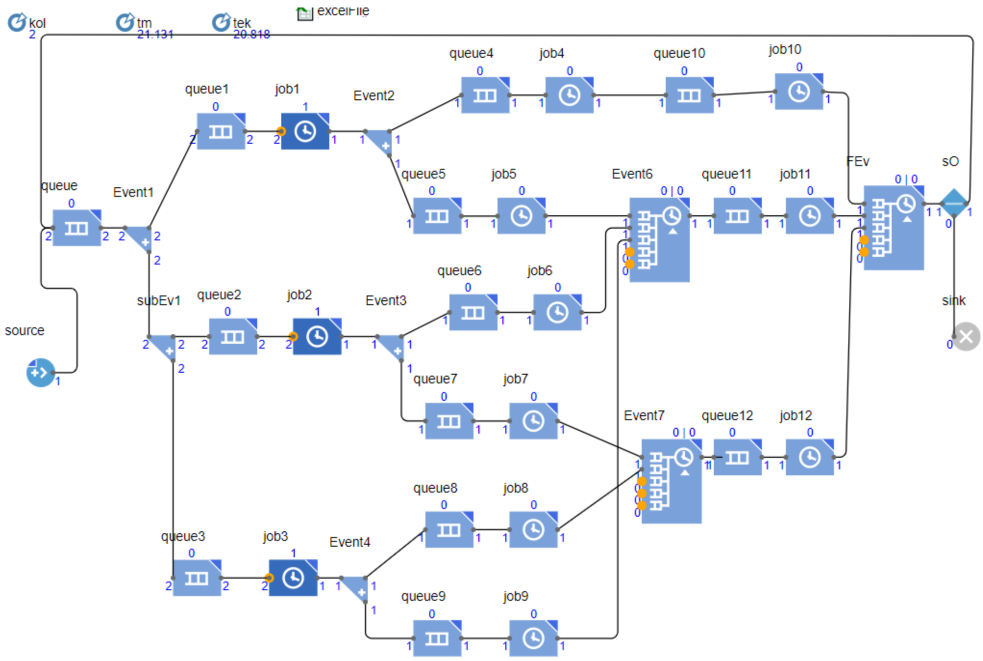

Fig. 5. The model of the project shown in Figure 1

The length of the critical path can be calculated using the formula (3) and is equal to 18 units (6 + 6 + 6). However, in the PERT method in this case, neither the number of critical paths nor the same asymmetry of jobs towards the highest value is taken into account. As a result of the run, the following project histogram was obtained (Figure 6).

Let's analyze the results obtained. First, it is obvious that the PERT method estimate for the critical path value in this case will be very rough. According to the histogram shown in Figure 6, the probability of completing a project within 18 units is extremely low. According to the experiment, it turned out that the expected time of the project, based on 6253 measurements, is equals to 20.168 units. In the figure, this value is schematically marked with a black line. Secondly, if the obtained distribution can be approximated with a certain degree of error by the normal distribution law, then the mathematical expectation (the parameter of the normal distribution showing the distribution center) of this law should be even greater than 20.168 (it is marked with a red line). It should be noted that this is an extreme case, when the project has a large number of critical paths and all activities have a distribution, the expected value of which is close to the maximum value. However, taking into account the possibility of different cases on real production or service projects, such situations must also be foreseen and an apparatus must be obtained that allows one to give more accurate estimates, which will be used in the future when forming the schedule.

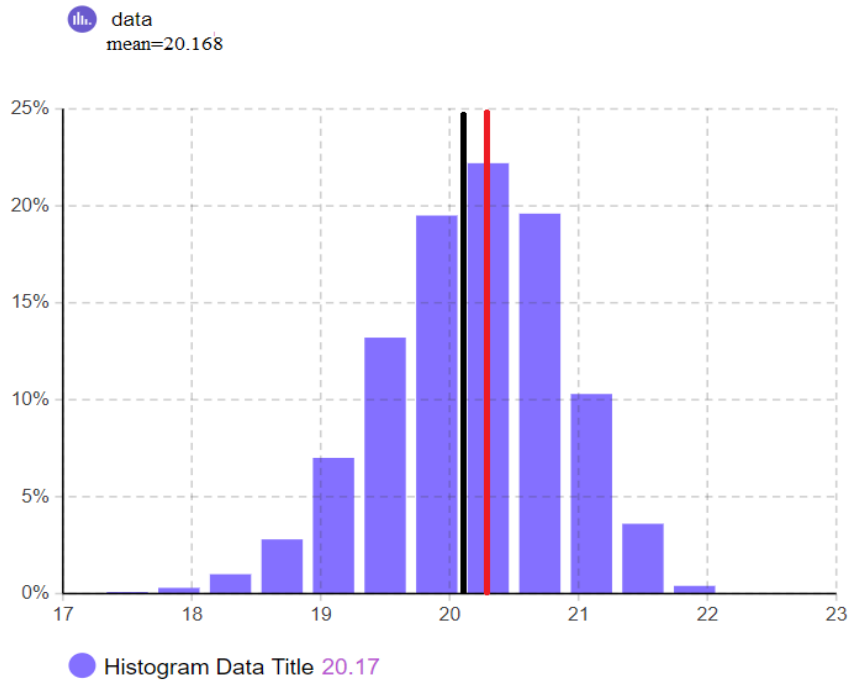

Fig. 6. Simulation results

4. CONCLUSION

The aim of the work was to build a simulation model that allows estimating the density of distribution of a random variable describing the duration of the project, as well as its numerical characteristics. For this, mechanisms are implemented that simulate the mutual dependence of jobs, as well as the ability to set the duration of each job, distributed according to the beta law with the specified parameters.

As a result, a simulation model of the project was obtained, shown in Fig. 6. Numerous computational experiments have shown that this model allows one to obtain an estimate of the distribution density of the investigated random variable, and its numerical characteristics, which are distinguished by increased accuracy relative to the common PERT method.

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