APPLICATION OF THE FUZZY ROBUST REGRESSION ANALYSIS, A CASE OF FUZZY DEPENDENT VARIABLE

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Abstract: The classic method of least squares is sensitive to outliers and other deviations from the basic initial data. In the presence of outliers in a data set, robust methods are better for estimating parameters. In this work, fuzzy robust regression is investigated when *x* are crisp and *Y* is a triangular fuzzy number and the outliers are present in the data set. A fuzzy robust regression method can detect outliers automatically. Due to the carried out researches the minimization of the negative impact of the outliers on the model has been achieved. Thanks to the improvement in the calculations of weight matrix the calculations of the parameters and the choice of the model for fuzzy robust regression were simplified. The obtained standardized covariance for the estimation of fuzzy robust regression model showed that the all parameters of the considered model are significant.

Key words: robust regression, fuzzy regression, membership, the weighted method of fuzzy least squares.

1. INTRODUCTION

If the statistical procedure is insensitive to large errors in the data, then it is called a robust. In such cases, it is preferable to use robust least squares methods. Fuzzy regression analysis is a fuzzy type of classical regression analysis that is used to find the functional relationship between dependent and independent variables in fuzzy events. Fuzzy robust regression methods have been successfully applied to various problems, such as prediction and engineering [1]. A combined robust and fuzzy least squares method was developed in this paper, when x is crisp and Y is a triangular fuzzy number, and outliers are present in the data set.

There was a need to reduce the impact of outliers on the model. The combined robust and fuzzy least squares method will determine outliers. It is also necessary to improve the calculation of parameters and the choice of a model for fuzzy robust regression.

The goal of this paper is to research and develop a unified method for estimating the robust and fuzzy least squares method, which minimizes the possible negative effects of emissions on the model.

2. ANALYSIS OF LITERATURE DATA AND PROBLEM STATEMENT

Fuzzy regression analysis of the model is divided into two methods.

The first one is based on the linear programming approximation [2], the second is based on the fuzzy least squares approximation proposed in [3]. In [4], a new approximation based on linear programming for calculating a fuzzy regression model was developed.

Observations that have larger residuals than others are called outliers. If outliers are in the data set, robust methods for estimating quantities are preferred.

In [2], a first linear regression analysis study with a fuzzy model was proposed. This approach was generalized in [5]. The approach developed in [2] is very difficult to solve the optimization problem. It is not clear what the attitude to the concept of least squares. Measurement of best fit is not presented.

The author in [3] proposed the fuzzy least squares method to eliminate those model defects in [2]. A model that often gave crisp numbers was criticized in [6]. In response to the criticism in [6], the authors improved the method of obtaining fuzzy interaction coefficients. Shown in [7], this method was highly sensitive to outliers, in [8] they proposed an interaction algorithm that is iterative to satisfy the solution. But this algorithm is very sensitive to outliers too. Through the method of orthogonal least squares in [7] improved the model proposed in [8], and also proposed a linear programming problem. In [9], a generalized method with a fuzzy weighted least squares method was proposed. For a simple regression, in [10], weighted fuzzy least squares of the iterative algorithms being analyzed were proposed. In solving of fuzzy regression method, weighted least squares were considered as an optimization problem. In [11], a linear regression analysis of fuzzy least squares was proposed for fuzzy input and fuzzy output. They used cluster analysis to determine emissions. In [12], an algorithm for fuzzy least squares was proposed for linear regression models of fuzzy interaction. This algorithm is resistant to outliers for simple regression. In this algorithm, orthogonal conditions added a constraint for the optimization problem. A review of fuzzy regression models is presented in [13, 14], but the methods are sensitive to outliers. Robust methods are presented in [15-19]. Fuzzy linear regression model is considered in [20]. Fuzzy robust regression model is presented in [21-24].

Consider a multiple regression model when the dependent variables are represented by triangular fuzzy numbers. Fuzzy robust regression is investigated when x is crisp and Y is a triangular fuzzy number, and outliers are present in the data set. To achieve the goal of this work it is necessary to solve the following tasks:

- to analyze the known methods of M-estimates (maximum likelihood estimates), smoothed-reduced M-estimates; to analyze the known fuzzy regression methods;
- to develop a combined robust and fuzzy regression method for the case when the independent variables are crisp, and the dependent variable is a triangular fuzzy number;
- for the case when the dependent variables are fuzzy, to obtain simulation results for comparing the methods of Huber, Hampel, Tukey, Andrews, Ψ-function, OLS and using the developed combined method of robust and fuzzy regression.

3. MATERIAL AND RESULTS OF THE RESEARCHES OF ROBUST REGRESSION, FUZZY REGRESSION, FUZZY ROBUST REGRESSION, SIMULATION RESULTS

In the work for the parameters of fuzzy robust regression, the estimates of standardized deviations are calculated through the weight matrix and hypothesis testing is applied to the parameters. Robust methods are given and fuzzy regression method is discussed. Hypothesis testing was performed for the LS method, M-methods — Huber, Hampel, Andrews, Tukey, Ψ -function, FRR method, and the numerical results of hypothesis testing are compared.

3. 1. Robust methods

M-estimation is based on the principle of replacing the squares of residuals used in OLS estimating, another function of residuals, to obtain:

$$\min_{\hat{\theta}} \sum_{i=1}^{n} \rho(r_i), \quad (1)$$

where ρ is a symmetric function with a minimum in zero, the properties of which are given below. Differentiating the sum of equation (1) by coefficient β_j and equating the partial derivatives to zero, we get:

$$\sum_{i=1}^{n} \psi\left(\frac{r_i}{d}\right) x_{ij} = 0,\ j = 1,2,\dots,p, \quad (2)$$

$$\psi(t) = \rho'(t), \quad r_i = y_i - \sum_{j=1}^{p} x_{ij}\hat{\beta}_j \quad (3)$$

where ψ is the derivative from ρ, x_i is the row vector of explanatory variables of the i-th observation. Solving the system of 'p' of nonlinear equations, we obtain an M-estimate. W-function (weight function) for any ρ is defined as

$$\omega(t_i) = \frac{\psi(t_i)}{t_i}. \quad (4)$$

Standardized residuals $t_i = \frac{r_i}{d}$. If r_i -the residuals of the i - th observation,

$$d = \frac{median|r_i|}{0{,}6745}, \quad i = 1,2,\ldots n \tag{5}$$

The weighted least squares method (WLS) is obtained using the W-functions in the OLS. The obtained estimates are weighted estimates. Weighted estimates are calculated by solving the equations, where W is a diagonal square matrix whose diagonal elements are weights:

$$\hat{\beta} = \left(X^T W X\right)^{-1} X^T W y. \tag{6}$$

The ψ -Huber function is defined as [14]:

$$\psi(t) = \begin{cases} -a, & t < -a, \\ t, & |t| \le a, \\ a, & t > a, \end{cases} \tag{7}$$

where a – the tuning constant ($a = 1{,}5$).

Reduced M-estimates. The reduced M-estimates were proposed by Hampel, he used three parts of the reduced estimates with ρ-functions. The bounded ψ-function is 0 for large | t | [18]. The three-part of reduced ψ-function is defined as:

$$\psi(t) = \begin{cases} sgn(t)|t|, & 0 \le |t| \prec a \\ a\, sgn(t), & a \le |t| \prec b \\ \left\{\frac{(c-|t|)}{(c-b)}\right\} a\, sgn(t), & b \le |t| \prec c \\ 0, & c \le |t| \end{cases} \tag{8}$$

where a=1,7; b = 3,4; c = 8,5.

When data contains outliers, robust covariance measures are required. The covariances of the estimated robust regression coefficients are the square roots of the diagonal components of the matrix of dimension $(p \times p)$:

$$\frac{\frac{d^2}{n-p}\sum(\psi(t_i))^2}{\left(\frac{1}{n}\sum \psi'(t_i)\right)^2}\left(X^T X\right)^{-1} \tag{9}$$

where ψ' is the derivative of ψ [15].

Various authors have proposed smoothly reduced M-estimates. An improved result was obtained in [13-15], who used wave estimates (sine estimates) and biweight estimator. They proposed a ψ-function with a weight beta function, where α = β [16, 17]. The wave function of Andrews is defined as [15]:

$$\psi(t) = \begin{cases} sin\left(\frac{t}{a}\right), & |t| \le \pi a \\ 0, & |t| > \pi a. \end{cases} \tag{10}$$

where $a = 2,1$. If the scale is known, then $a = 1,339$ that corresponds to a 5% loss of efficiency. Tukey biweight function [13, 14]:

$$\psi(t) = \begin{cases} t\left[1-\left(\frac{t}{a}\right)^2\right]^2, & |t| \le a, \\ 0, & |t| > a, \end{cases} \tag{11}$$

where $a = 6,0$. If the scale is known, then $a = 4,685$ that corresponds to a 5% loss of efficiency.

New ψ-function representation. A new representation of the ρ-function was proposed in the family of smoothly reduced (smoothly redescending) M-estimates [18]. Due to the increased efficiency, the ψ-function, associated with this ρ-function, has a greater linearity in the central part compared to other ψ-functions - the Andrews sine, Tukey biweight and the Kadir beta-function. Multiple weighted (reweighted) least squares method (IRLS) [19], based on the proposed ρ-function, detects outliers and specifies the presence of outliers in the subsequent analysis. The method gives improved results in all situations, can hold a significant amount of outliers.

Suggested ρ-function:

$$\rho(t) = \begin{cases} \frac{t^2}{45a^8}\left(3t^8 - 10a^4t^4 + 15a^8\right), & if\ |t| \le a, \\ \frac{8a^2}{45}, & if\ |t| \succ a, \end{cases} \tag{12}$$

The proposed ψ-function [18] is given below.

$$\psi(t) = \begin{cases} \frac{2t}{3}\left(1-\left(\frac{t}{a}\right)^4\right)^2, & if\ |t| \le a, \\ 0, & if\ |t| \succ a, \end{cases} \tag{13}$$

where *a* is the tuning constant for the *i*-th observation, the variable t is the residuals scaled by MAD (median of the absolute deviations), ρ is the function that corresponds to the ψ-function given above, satisfies the standard properties. They are associated with a reasonable objective function.

3. 2. Fuzzy regression analysis

Triangular fuzzy numbers are defined as $X = (m, \underline{m}, \overline{m})$, where m is the central value, $\underline{m}, \overline{m}$ are left and right variations.

When $X_i = \left(x_i, \underline{\xi}_i, \overline{\xi}_i\right)$ and $Y_i = \left(y_i, \underline{\eta}_i, \overline{\eta}_i\right), i = 1,2,\ldots n$ are triangular fuzzy numbers, the regression model is given by the equation:

$$Y = a + bX \tag{14}$$

where a, b are crisp numbers.

The optimization problem for fuzzy least squares is defined as:

$$minimize\ \ r(a,b) = \sum d(a + bX_i, Y_i)^2 \tag{15}$$

$$d(a+bX_i, Y_i)^2 = \left[a + bx_i - y_i - \left(b\underline{\xi}_i - \underline{\eta}_i\right)\right]^2 + \left[a + bx_i - y_i + \left(b\overline{\xi}_i - \overline{\eta}_i\right)\right]^2 + \left(a + bx_i - y_i\right)^2. \tag{16}$$

The parameters a, b are fined from equalities $\frac{\partial r}{\partial a} = 0$ and. $\frac{\partial r}{\partial b} = 0$.

The fuzzy least squares model is represented by a generalized multidimensional model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p + \varepsilon$. In this case, the optimization problem is defined as:

$$Min\ r(a, b_1, b_2, \ldots, b_p) = \sum d(a + b_1 X_{1i} + \ldots + b_p X_{pi}, Y_i)^2 \tag{17}$$

The parameters are determined from the minimization of equation (17). The iterative procedure is continued until the required degree of convergence of the process will be achieved. For $i = 1, 2, \ldots, n$ when x_i a crisp and $Y_i = (y_i, \underline{\eta}_i, \overline{\eta}_i)$ - triangular fuzzy number, a fuzzy regression model

$$Y = A + xB \tag{18}$$

where A=$(a, \underline{\alpha}, \overline{\alpha})$, $B = (b, \underline{\beta}, \overline{\beta})$ - fuzzy parameters.

The parameters are estimated through an equation that presents the fuzzy least squares optimization problem:

$$Minimize\ \ r(A,B) = \sum d(A + x_i B, Y_i)^2 \quad (19$$

$$d(A + x_i B, Y_i)^2 = (a + bx_i - y_i)^2 + (a + bx_i - \underline{\alpha} - \underline{\beta} x_i - y_i + \underline{\eta}_i)^2 + + (a + bx_i + \overline{\alpha} + x_i \overline{\beta} - y_i - \overline{\eta}_i)^2, \tag{20}$$

where for $\frac{\partial r}{\partial a} = 0$ and $\frac{\partial r}{\partial b} = 0$ the parameters a and b are determined, for $\frac{\partial r}{\partial \alpha} = 0$ and $\frac{\partial r}{\partial \beta} = 0$ the parameters α and β are calculated [4, 20, 21].

3.3. Fuzzy robust regression analysis

For a multidimensional model, the parameters are estimated through the equation:

$$Min\ r(A, B_1, B_2, \ldots, B_P) = \sum d(A + B_1 x_{1i} + \cdots B_P x_{Pi}, Y_i)^2$$

The median is determined with respect to the absolute values of the residuals, and the distances D_i are calculated:

$$D_i = \left\| abs(r_i) - median(abs(r_i)) \right\|, i = 1, 2, \ldots, n,$$

where || || is the Euclidean distance [22, 23].

The membership function is defined as:

$$\mu(r)=\begin{cases} 1, & |r|\le a \\ \dfrac{b-|r|}{b-a}, & a \prec |r| \prec b \\ 0, & otherwise, \end{cases} \tag{21}$$

where $a = median(D_i)$, $b = max(D_i) + d$, $d = \dfrac{median|r_i|}{0{,}6745}$, $W = diag(\mu_1, \mu_2, \ldots, \mu_n)$

The membership function is determined from equation (21), the membership function values are determined, and a weight matrix is constructed. A weight matrix is a diagonal matrix with diagonal elements of the membership function. Weighted estimates of fuzzy least squares are determined using the resulting weight matrix. If $\left|\hat{\beta}^{k+1} - \hat{\beta}^{k}\right| \prec \varepsilon$ then stop, otherwise return to the calculation $\hat{y}_i$ and r_i . The function ψ is defined in relation to the weight function [24] for fuzzy robust regression (FRR):

$$\psi=\begin{cases} |r|, & |r|\le a \\ \dfrac{|r|b-|r|^2}{b-a}, & a \prec |r| \prec b \\ 0, & otherwise. \end{cases} \tag{22}$$

Thanks to performed modifications in the calculations of the weight matrix the calculation of parameters and the choice of a model for fuzzy robust regression are simplified.

3.4. Results of experiments

Let's analyze the effect of Portland cement on heat generation during hardening. An example containing 13 observations, three independent variables and one dependent variable [21] is considered and hypotheses are tested for parameters. The eighth observation is an outlier. The data set is presented in Table 1. M- methods and fuzzy robust regression method are investigated by example. Estimates of the regression coefficients and the covariance of the estimated coefficients are presented in Table 2. It is seen that the covariance for the ψ- function of the M-method are less than the covariance for the corresponding parameters of the other M -methods. For an example with a data set containing 13 observations and 3 independent variables for $\alpha = 0.05$ the value t is 2.262. For each parameter, the calculated values are higher than this value. Therefore, we reject the null hypothesis with significance level α. In accordance with this, all parameters are significant. Values η_i are assumed by the author.

Table 1. Data set

I	x_1	x_2	x_3	(y_i, η_i)
1	7	26	6	(78.5, 6.9)
2	1	29	15	(74.3, 6.4)
3	11	56	8	(104.3, 9.4)
4	11	31	8	(87.6, 7.8)
5	7	52	6	(95.5, 8.6)
6	11	55	9	(109.2, 9,9)
7	3	71	17	(102.7, 9.3)
8	1	31	22	(60, 6.2)
9	2	54	16	(93.3, 8.3)
10	21	47	4	(115..9, 10.6)
11	1	40	23	(83.8, 7.4)
12	11	66	9	(113.3, 10.6)
13	10	68	8	(109.4, 9.9)

Here: y_i the heat released in calories per gram of cement; x_{i1} amount of tricalcium aluminate; x_{i2} amount of tricalcium silicate; x_{i3} amount of tetracalcium alumoferrite; x_{i1}, x_{i2}, x_{i3} are measured as a percentage from the weight of the clinker from which the cement was made.

Table 2. Estimates of regression coefficients and the covariance of the coefficients

Method	constant	Regression coefficients		
		β_1	β_2	β_3
LS	49.4442	1.5253	0.7248	-0.1087
	(8.5282)	(0.4458)	(0.0964)	(0.4025)
Huber	47.9607	1.7021	0.6557	0.2641
	(4.4247)	(0.2313)	(0.0500)	(0.2088)
Hampel	47.3509	1.7757	0.6248	0.4267
	(3.0715)	(0.1606)	(0.0347)	(0.1450)
Tukey	47.4435	1.7733	0.6236	0.4256
	(3.1734)	(0.1658)	(0.0359)	(0.1498)
Andrews	47.3268	1.7810	0.6255	0.4276
	(3.2869)	(0.1718)	(0.0372)	(0.1581)
Diamond	(49.4442, 3.7962)	(1.5253, 0.1704)	(0.7248, 0.0668)	(-0.1087, 0.0234)
ψ -function	47.8136	1.7509	0.6158	0.4214
	(2.7278)	(0.1426)	(0.0308)	(0.1287)
Fuzzy Robust Regression	(47.4157, 3.7376)	(1.7693, 0.1778)	(0.6309, 0.0638)	(0.4018, 0.0395)
	(2.5814)	(0.1348)	(0.0294)	(0.1217)

The covariances for the estimated fuzzy robust regression model are obtained through the robust covariance defined in equation (9).

Results of computer modeling using developed methods are presented in Fig. 1.

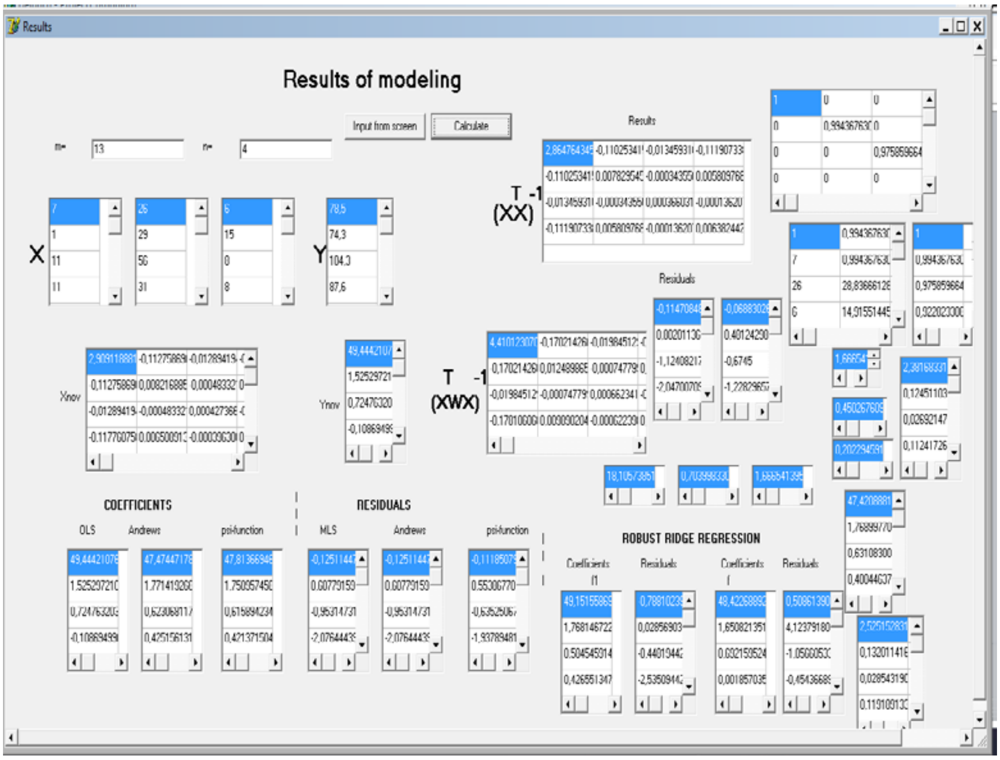

Fig. 1. Results of computer modeling using developed methods

4. DISCUSSION OF THE RESULTS OF THE ROBUST AND FUZZY REGRESSION RESEARCHES, RESULTS OF EXPERIMENTS

A combined robust and fuzzy regression (FRR) method has been developed for the case when the independent variables are crisp and the dependent variable is a symmetrical triangular fuzzy number. Fuzzy robust multiple regression is robust for estimating a fuzzy regression model and with outliers. This is the advantage of the developed method. This method automatically detects outliers. The weight matrix is obtained by the fact that each observation is included in the estimation of the regression model, depending on the degree of belonging.

The impact of outliers on the model is minimized. The advantage is to find observations for further study. As a result of the research the null hypothesis with the significance level α is rejected. In accordance with this, all parameters of the considered model are significant. Thanks to modifications in the calculations of the weight matrix, the calculation of parameters and the choice of a model for a fuzzy robust regression are simplified.

Fuzzy robust regression methods are successfully used in various fields: economics, communications, astronomy, meteorology.

5. CONCLUSIONS

As a result of the research the analysis of known methods is carried out. A combined robust and fuzzy regression (FRR) method has been developed for the case when the independent variables are crisp, and the dependent variable is a symmetric triangular fuzzy number.

1. The analysis of the known methods of M-estimates (maximum likelihood estimates), smoothly reduced M-estimates; analysis of known fuzzy regression methods are carried out.

2. A combined robust and fuzzy regression method has been developed for the case when the independent variables are crisp and the dependent variable is a triangular fuzzy number;

3. For the case when the dependent variables are fuzzy, simulation results are obtained comparing the methods of Huber, Hampel, Tukey, Andrews, ψ -function, OLS and using the developed combined method of robust and fuzzy regression. A simulation was carried out using an example containing 13 observations, three independent variables and one dependent variable, and a hypothesis test was made for the parameters, where the eighth observation is an outlier. The M-methods and the fuzzy robust regression method are investigated by example. Estimates of the regression coefficients and the covariance of the estimated coefficients are presented in this paper, for an example with a data set containing 13 observations and 3 independent variables for $\alpha = 0.05$ $t = 2.262$. For each parameter, the calculated values are higher than this value t. Therefore, the null hypothesis with the level of significance α was rejected. In accordance with this, all parameters are significant. A fuzzy robust regression method helped determine outliers. Thus, the possible negative impact of the outliers on the model and in comparison with other M-methods is minimized. In accordance with the standardized covariances obtained for the estimated fuzzy robust regression model, all parameters of the considered model are significant. Calculation of parameters and model selection for fuzzy robust regression is improved.

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