NUMERICAL SOLUTION AND EFFECTIVE ERROR ESTIMATION FOR A NONLOCAL MIXED PROBLEM FOR THE LAPLACE EQUATION IN THE PROCESS OF INTELLIGENT MODELING

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Abstract: In this paper, the error of the discrete Fourier method for non-local mixed problem is effectively estimated based on a known data obtained in the process of intelligent modeling. Algorithms and the software were developed to solve the problems posed by the discrete analogue of the Fourier method. Numerical experiments were conducted which have confirmed the reliability of the results obtained.

Key words: Laplace equation, non-local mixed problem, discrete Fourier method, difference scheme, approximate solution.

1. INTRODUCTION

By applying the grid method for the error estimates to approximate solution of the Laplace equation usually contains maximums of derivatives in modules of the desired solution. Moreover, this naturally makes difficulty to use estimates in practice in the process of making informed decisions of the desired outcomes of the tasks being solved. Error estimates of some methods expressed by considered problem data are known from the references [1-6] and became the considered for the knowledge base in the formation of an appropriate expert decision-making system [7, 8].

The following non-local mixed problem is considered. It is required to find a solution of the equation

$$\Delta u = 0 \text{ on } \Pi \tag{1}$$

satisfying the boundary conditions

$$\frac{\partial u}{\partial x} = 0 \text{ on } \Gamma_2, \Gamma_4, \tag{2}$$

$$u = \varphi \text{ on } \Gamma_3, \tag{3}$$

$$u(x,0) = \alpha\, u(x,c), (\alpha < 1) \tag{4}$$

where it is assumed that $\varphi(x)$ is thrice continuously differentiable and

$$\varphi'(0) = \varphi'(1) = 0.$$

Here $\Pi = \{(x,y): 0 < x < 1, 0 < y < b\}$, and $\Gamma_i \ (i = \overline{1,4})$ are the sides of the rectangular Π numbered counterclockwise starting with lower-side except for the ends. Introduce the following notation: $\Gamma = \bigcup_{i=1}^{4} \Gamma_i, \overline{\Pi} = \Pi \bigcup \Gamma$.

Introduce quadratic net by straight lines $x = x_i = ih, \ y = y_j = jh \quad (i, j = \overline{1,n})$. Denote $\Pi_h = \{(x,y): x = x_i = ih, y = y_j = jh, i = \overline{0,n}\}$, $\Gamma_{ih} \ (i = \overline{1,4})$ is a set of net nodes lying on Γ_i and $\Gamma_h = \bigcup_{i=1}^{4} \Gamma_{ih}$ respectively, as well as $\overline{\Pi}_h = \Pi_h \bigcup \Gamma_h$.

2. DIFFERENCE SCHEME

Let's construct the difference scheme of corresponding problem (1)-(4) in the following form

$$\Delta_h u_h = 0 \text{ on } \Pi_h, \tag{5}$$

$$-\frac{2}{h} u_{\bar{x}} + u_{\bar{y}y} = 0 \text{ on } \Gamma_{2h}, \ \frac{2}{h} u_{\bar{x}} + u_{\bar{y}y} = 0 \text{ on } \Gamma_{4h}, \tag{6}$$

$$u_h = \varphi_h \text{ on } \Gamma_{3h}, \tag{7}$$

$$u_h(x,0) = \alpha u_h(x,c), \ (\alpha < 1). \tag{8}$$

Here it is assumed that the point c coincides with one of nodes.

We prove that the solutions of problem (1)-(4) and (5)-(8) are defined by the following formulae respectively

$$u(x,y) = \frac{\alpha c + (1-\alpha)y}{2[\alpha c + (1-\alpha)b]} a_0 + \sum_{n=1}^{\infty} a_n g(y, n\pi) \cos n\pi x, \tag{9}$$

$$u_h(x,y) = \frac{\alpha c + (1-\alpha)y}{2[\alpha c + (1-\alpha)b]} b_0 + \sum_{n=1}^{1/h} \beta_n g(y, \beta_n / h) \cos n\pi x, \tag{10}$$

$$a_n = 2\int_0^1 \varphi(t) \cos n\pi t dt, \ a_0 = \int_0^1 \varphi(t) dt, \quad b_n = 2h \sum_{k=1}^{1/h} \varphi_h(kh) \cos n\pi kh, \ b_0 = h \sum_{k=1}^{1/h} \varphi_h(kh),$$

$$g(y,z) = \frac{sh\, yz - \alpha sh(y-c)z}{sh\, bz - \alpha sh(b-c)z}, \tag{11}$$

$$sh \frac{\beta_n}{2} = \sin \frac{nh\pi}{2}.$$

At first we prove formula (9). We'll search a solution in the form $u(x,y) = X(x)Y(y)$. Then

$$X'' - kX = 0, \tag{12}$$

$$Y'' + kY = 0 \tag{13}$$

where k is some constant.

In order the function $u(x,y)$ satisfy boundary conditions (2) the function $X(x)$ must satisfy the conditions

$$X'(0) = X'(1) = 0. \tag{14}$$

We must find a solution of equation (12) satisfying condition (14). If assume $k = -\lambda^2$, then the general solution of equation (12) will be

$$X(x) = C_1 \cos\lambda x + C_2 \sin\lambda x.$$

Condition (14) gives $X'(0) = \lambda C_2 = 0, \quad X'(1) = -\lambda C_1 \sin\lambda + \lambda C_2 \cos\lambda = 0.$

Consequently, $C_2 = 0$ certainly and we can take C_1 being equal to zero only provided $\sin\lambda = 0$, i.e. if λ is a number divisible by π : $\lambda = n\pi$ $(n = 1,2,...)$.

At $\lambda = n\pi$ we obtain the solution $X(x) = C_1 \cos n\pi x$ $(n = 1,2,...)$.

Substituting $K = -\lambda^2 = -(n\pi)^2$ in (13) $Y'' - (n\pi)^2 Y = 0$ we obtain for Y the equation $Y'' - (n\pi)^2 Y = 0$ whose solution is defined by the formula $Y = C_1 ch\, n\pi y + C_2 sh\, n\pi y$ where C_1, C_2 are arbitrary constants

$$u(x,y) = \sum_{n=1}^{\infty} [A_n ch\, n\pi y + B_n sh\, n\pi y] \cos n\pi x + A_0 y + B_0.$$

This function satisfies equation (1) and boundary conditions (2), it remains to select the constants A_n and B_n such that to satisfy boundary conditions (3) and (4). According to these solutions

$$\sum_{n=1}^{\infty} [A_n ch\, n\pi b + B_n sh\, n\pi b] \cos n\pi x + A_0 b + B_0 = 0,$$

$$\sum_{n=1}^{\infty} A_n \cos n\pi x + B_0 = \alpha \left\{ \sum_{n=1}^{\infty} [A_n ch\, n\pi c + B_n sh\, n\pi c] \cos n\pi x + A_0 c + B_0 \right\} + \varphi.$$

On the other hand, subject to $\varphi(t) = \sum_{n=1}^{\infty} a_n \cos n\pi t + \frac{a_0}{2}$, where

$$a_n = 2\int_0^1 \varphi(t) \cos n\pi t dt, \; a_0 = 2\int_0^1 \varphi(t) dt \cdot$$

Hence comparing the coefficients of series, we obtain

$$A_0 = \frac{a_0}{2[\alpha c + (1-\alpha) b]}, \quad B_0 = \frac{b_0}{2[\alpha c + (1-\alpha) b]},$$

$$A_n = \frac{sh\; n\pi b}{(1 - \alpha ch\, n\pi c) sh\, n\pi b + \alpha sh\, n\pi c\; ch\, n\pi b} a_n,$$

$$B_n = \frac{ch\; n\pi b}{(1 - \alpha ch\, n\pi c) sh\, n\pi b + \alpha sh\, n\pi c\; ch\, n\pi b} a_n.$$

Thus $u(x,y) = \frac{\alpha c + (1-\alpha) y}{2[\alpha c + (1-\alpha) b]} a_0 + \sum_{n=1}^{\infty} a_n \frac{sh\; n\pi y - \alpha sh(y-c) n\pi}{sh\; n\pi b - \alpha sh(b-c) n\pi} \cos n\pi x \cdot$

By immediate testing it easy to be convinced that $u_h(x,y)$ defined by formula (10) is a solution of problem (5)-(8). It is known that [2] $a_n = b_n, (n = 0,1,2,,,1/h)$. We'll get the solution of difference scheme (5)-(8) $u_h(x,y)$ as a approximate solution of problem (1)-(4).

3. ESTIMATE ERROR OF METHOD

We estimate error of method. From (9) and (10) we have $|u - u_h| \le R_1 + R_2$, where $R_1 = \sum_{n=1}^{1/h} |b_n| \left| g(y, n\pi) - g(y, \frac{\beta_n}{h}) \right|$, $\quad R_2 = \sum_{n=1+1/h}^{\infty} |a_n| \|g(y, n\pi b)|$.

Consequently, in order to estimate $R_i\,(i = 1,2)$ it is necessary to estimate $|g(y,z)|$ and $\left| g(y, n\pi) - g(y, \frac{\beta_n}{h}) \right|$. It is easy to note that $0 \le g(y,z) \le 1$.

We estimate $\left| g(y, n\pi) - g(y, \frac{\beta_n}{h}) \right|$. Considering

$$(b+y)sh(b-y)z \le (b-y)sh(b+y)z,$$
$$(b-c+y)sh(b-c-y)z \le (b-c-y)sh(b-c+y)z$$

and in $y \ge c$,

$$(b-c+y)sh(b-(y-c))z \le (b+c-y)sh(b+y-c)z,$$
$$(b+y-2c)sh(b-y)z \le (b-y)sh(b-y+2(y-c))z,$$

in $y \le c$,

$$(b-c+y)sh(b+c-y)z \ge (b+c-y)sh(b-(c-y))z,$$
$$(b+y-2c)sh(b-y)z \ge (b-y)sh(b-y-2(c-y))z.$$

We obtain

$$\left|\frac{\partial g(y,z)}{\partial z}\right| \le \frac{1}{2}[sh\,bz - \alpha\, sh(b-c)z]^{-2}\{(b-y)sh(b+y)z + \\ +\alpha(b-c-y)sh(b-c+y)z + (b-|y-c|)sh(b+|y-c|)z + \\ +\alpha(b-c-|y-c|)sh(b-c+|y-c|)z\}. \tag{15}$$

We'll use below the following obvious inequalities

$$sh\,kt \le \exp((k-1)t)sh\,t, \quad (0 \le k \le 1), \quad sh\,t \ge \frac{1}{2}(1-\exp(-2t_1))\exp(t), \ (t \ge t_1 > 0).$$

Hence, we obtain

$$\left|\frac{\partial g}{\partial z}\right| \le 2\left[1-\exp\left(-\frac{8b}{3}\right) - \alpha\exp\left(-\frac{4c}{3}\right)\right]^{-2} \exp(2bz)sh\,2bz\{(b-y)\exp(-(b-y)z) + \\ \alpha(b-c-y)\exp(-(b+c-y)z) + (b-|y-c|)\exp(-(b-|y-c|)z) + \alpha(b-c-|y-c|)\times \\ \times\exp(-(b+c-|y-c|)z)\}.$$

Considering $\exp(-2bz)sh\,2bz \le \frac{1}{2}$, we obtain

$$\left|\frac{\partial g}{\partial z}\right| \le \left[1-\exp\left(-\frac{8b}{3}\right) - \alpha\exp\left(-\frac{4c}{3}\right)\right]^{-2} \{(b-y)\exp(-(b-y)z) + \alpha(b+c-y)\times \\ \times\exp(-(b+c-y)z) + (b-|y-c|)\exp(-(b-|y-c|)z) + \\ \alpha(b+c-|y-c|)\exp(-(b+c-|y-c|)z)\}.$$

Thus at $\frac{4}{3}n \le \frac{\beta_n}{h} < z < n\pi, 0 \le y \le b, 1 \le n \le 1/h$ we obtain

$$\left|g\left(y,\frac{\beta_n}{h}\right)-g(y,n\pi)\right| \le \left[1-\exp\left(-\frac{8b}{3}\right)-\alpha\exp\left(-\frac{4c}{3}\right)\right]^{-2}\left\{(b-y)\exp\left(-(b-y)\frac{4}{3}n\right)+\right.$$

$$+\alpha(b+c-y)\exp\left(-(b+c-y)\frac{4}{3}n\right)+(b-|y-c|)\exp\left(-(b-|y-c|)\frac{4}{3}n\right)+$$

$$\left.+\alpha(b+c-|y-c|)\exp\left(-(b+c-|y-c|)\frac{4}{3}n\right)\right\}\frac{(n\pi)^3}{6}h^2.$$

Estimate R_1:

$$R_1=\sum_{n=1}^{1/h}|b_n|\left|g\left(y,\frac{\beta_n}{h}\right)-g(y,n\pi)\right| \le K\frac{(1+\alpha)}{4}\pi^3h^2\left(1-\exp\left(-\frac{8b}{3}\right)-\alpha\exp\left(-\frac{4c}{3}\right)\right)^{-2}.$$

Thus $|u-u_h| \le K\dfrac{h^2}{3}+\dfrac{1+\alpha}{8}Kh^2\pi^3\left(1-\exp\left(-\dfrac{8b}{3}\right)-\alpha\exp\left(-\dfrac{4c}{3}\right)\right)^{-2}$,

$$|u-u_h| \le K\left\{\frac{1}{3}+\frac{1+\alpha}{4}\pi^3\left(1-\exp\left(-\frac{8b}{3}\right)-\alpha\exp\left(-\frac{4c}{3}\right)\right)^{-2}\right\}h^2.$$

Now consider the following problem. It is required to find a solution of the equation

$$\Delta u=0 \text{ on } \Pi \tag{16}$$

satisfying the boundary conditions

$$\frac{\partial u}{\partial y}=0 \text{ on } \Gamma_3,\ \Gamma_4, \tag{17}$$

$$u=\varphi(y) \text{ on } \Gamma_2, \tag{18}$$

$$u(0,y)=\alpha u(c,y),\ (\alpha<1) \text{ on } \Gamma_1, \tag{19}$$

where it is assumed that $\varphi(y)$ is thrice continuously differentiable and

$$\varphi'(0)=\varphi'(b)=0.$$

Let's construct the difference scheme of corresponding problem (16)-(19) in the following form

$$\Delta_h u_h=0 \text{ on } \Pi_h, \tag{20}$$

$$-\frac{2}{h}u_{\bar{x}}+u_{\bar{y}y}=0 \text{ on } \Gamma_{3h},\ \frac{2}{h}u_{\bar{x}}+u_{\bar{y}y}=0 \text{ on } \Gamma_{4h}, \tag{21}$$

$$-\frac{2}{h}u_{\bar{x}}+u_{\bar{y}y}=\varphi_h(y) \text{ on } \Gamma_{2h}, \tag{22}$$

$$u_h(0,y)=\alpha u_h(c,y),\ (\alpha<1) \text{ on } \Gamma_{1h}, \tag{23}$$

Here it is assumed that the point c coincides with one of nodes.

We prove that the solutions of problem (16)-(19) and (20)-(23) are defined by the following formulae respectively

$$u(x,y)=\frac{a_0}{2(1-\alpha)}+\sum_{n=1}^{\infty}a_n g(x,n\pi)\cos\frac{n\pi y}{b}, \tag{24}$$

$$u_h(x,y)=\frac{b_0}{2(1-\alpha)}+\sum_{n=1}^{1/h}b_n g(x,\beta_n/h)\cos\frac{n\pi y}{b}, \tag{25}$$

$$a_n = \frac{2}{b}\int_0^b \varphi(t)\cos\frac{n\pi t}{b}dt,\ a_0 = \int_0^b \varphi(t)dt,\ b_n = \frac{2h}{b}\sum_{k=1}^{1/h}\varphi_h(kh)\cos\frac{n\pi kh}{b},\ b_0 = \frac{h}{b}\sum_{k=1}^{1/h}\varphi_h(kh),$$

$$g(x,z) = \frac{sh\frac{z}{b}x - \alpha sh\frac{z}{b}(x-c)}{ch\frac{z}{b} - \alpha ch\frac{z}{b}(1-c)}, \tag{26}$$

$$sh\frac{\beta_n}{2b} = \sin\frac{nh\pi}{2b}.$$

Similarly, to the previous problem, we find

$$u(x,y) = \sum_{n=1}^{\infty}[A_n ch\frac{n\pi x}{b} + B_n sh\frac{n\pi x}{b}]\cos\frac{n\pi y}{b} + A_0 x + B_0 \cdot$$

This function satisfies equation (16) and boundary conditions (17), it remains to select the constants A_n and B_n such that to satisfy boundary conditions (18) and (19). According to these solutions

$$\sum_{n=1}^{\infty}\frac{n\pi}{b}[A_n sh\frac{n\pi}{b} + B_n ch\frac{n\pi}{b}]\cos\frac{n\pi y}{b} + A_0 = \varphi(y),$$

$$\sum_{n=1}^{\infty}A_n \cos\frac{n\pi y}{b} + B_0 = \alpha\left\{\sum_{n=1}^{\infty}[A_n ch\frac{n\pi c}{b} + B_n sh\frac{n\pi c}{b}]\cos\frac{n\pi y}{b} + A_0 c + B_0\right\}$$

On the other hand, subject to $\varphi(t) = \sum_{n=1}^{\infty}a_n \cos\frac{n\pi t}{b} + \frac{a_0}{2}$, where

$$a_n = \frac{2}{b}\int_0^b \varphi(t)\cos\frac{n\pi t}{b}dt,\ a_0 = \int_0^b \varphi(t)dt\ .$$

Hence comparing the coefficients of series, we obtain

$$A_0 = \frac{a_0}{2}\ ,\ \frac{n\pi}{b}\left(A_n sh\frac{n\pi}{b} + B_n ch\frac{n\pi}{b}\right) = a_n\ ,$$

$$\sum_{n=1}^{\infty}\left\{A_n - \alpha\left[A_n ch\frac{n\pi c}{b} + B_n sh\frac{n\pi c}{b}\right]\right\}\cos\frac{n\pi y}{b} + B_0 - \alpha(A_0 c + B_0) = 0\ ,$$

$$B_0 - \alpha(A_0 c + B_0) = 0\ ,\ \left(1 - \alpha ch\frac{n\pi c}{b}\right)A_n - \alpha B_n sh\frac{n\pi c}{b} = 0$$

$$B_n = \frac{\left(1 - \alpha ch\frac{n\pi c}{b}\right)}{\alpha sh\frac{n\pi c}{b}}A_n,\quad B_0 - \alpha\left(\frac{a_0}{2}c + B_0\right) = 0,\quad B_0 = \frac{\alpha c}{2(1-\alpha)}a_0,$$

Thus

$$u(x,y) = \sum_{n=1}^{\infty}a_n\frac{\alpha\left(sh\frac{n\pi c}{b}ch\frac{n\pi x}{b} - ch\frac{n\pi c}{b}sh\frac{n\pi x}{b}\right) + sh\frac{n\pi x}{b}}{\alpha\left(sh\frac{n\pi}{b}sh\frac{n\pi c}{b} - ch\frac{n\pi c}{b}ch\frac{n\pi}{b}\right) + ch\frac{n\pi}{b}}\frac{b}{n\pi}\cos\frac{n\pi y}{b} + \frac{\alpha c}{2(1-\alpha)}a_0,$$

$$u(x,y)=\frac{\alpha c}{2(1-\alpha)}a_0+\sum_{n=1}^{1/h}a_n\frac{b}{n\pi}\frac{sh\frac{n\pi x}{b}+\alpha sh\frac{n\pi}{b}(x-c)}{ch\frac{n\pi}{b}-\alpha ch\frac{n\pi}{b}(1-c)}\cos\frac{n\pi y}{b}\cdot$$

The validity of the formula (24) is proved. Now we prove that $u_h(x,y)$ defined by (25) is the solution of the problem (20)-(23):

$$ch\frac{\beta_n}{b}+\cos\frac{nh\pi}{b}-2=0,\quad sh\frac{\beta_n}{2b}=\sin\frac{nh\pi}{2b}.$$

We'll get the solution of difference scheme (20)-(23) $u_h(x,y)$ as a approximate solution of problem (16)-(19).

We estimate error of method. From (24) and (25) we have $|u-u_h|\le R_1+R_2$, where

$$R_1=\sum_{n=1}^{1/h}|b_n|\left|g(x,n\pi)-g\left(x,\frac{\beta_n}{h}\right)\right|,\qquad R_2=\sum_{n=1+1/h}^{\infty}|a_n||g(x,n\pi)|.$$

Consequently, to estimate R_i $(i=1,2)$ it is necessary to estimate $|g(x,z)|$ and $\left|g(x,n\pi)-g\left(x,\frac{\beta_n}{h}\right)\right|$. It is easy to note that $0\le g(x,z)\le 1$.

We estimate $\left|g(x,n\pi)-g\left(x,\frac{\beta_n}{h}\right)\right|$. We have

$$\frac{\partial g}{\partial z}=\frac{1}{2}\left[ch\frac{z}{b}-\alpha ch\frac{z}{b}(1-c)\right]^{-2}\left\{\left(\frac{1}{b}+x\right)sh(1-x)\frac{z}{b}-\left(\frac{1}{b}-x\right)ch(1+x)\frac{z}{b}-\right.$$
$$-\alpha\left[(1-c+x)sh(1-c-x)\frac{z}{b}-(1-c-x)sh(1-c-x)\frac{z}{b}\right]+$$
$$+(1-c+x)sh(1+c-x)\frac{z}{b}-(1+c-x)sh(1-c+x)\frac{z}{b}-$$
$$\left.-\alpha\left[(1+x-2c)sh(1-x)\frac{z}{b}-(1-x)sh(1+x-2c)\frac{z}{b}\right]\right\}.$$

Considering

$$\left(\frac{1}{b}+x\right)sh(1-x)\frac{z}{b}\le\left(\frac{1}{b}-x\right)sh(1+x)\frac{z}{b},\ (1-c+x)sh(1-(c-x))\frac{z}{b}\le$$
$$\le(1-c-x)sh(1-c+x)\frac{z}{b}$$

and in $x\ge c$

$$(1-c+x)sh(1-(x-c))\frac{z}{b}\le(1+c-x)sh(1+x-c)\frac{z}{b},$$
$$(1+x-2c)sh(1-x)\frac{z}{b}\le(1-x)sh(1-x+2(x-c))\frac{z}{b},$$

in $x\le c$

$$(1-c+x)sh(1+c-x)\frac{z}{b} \geq (1+c-x)sh(1-(c-x))\frac{z}{b},$$

$$(1+x-2c)sh(1-x)\frac{z}{b} \geq (1-x)sh(1-x-2(c-x))\frac{z}{b}.$$

We obtain

$$\left|\frac{\partial g(x,z)}{\partial z}\right| \leq \frac{1}{2}\left[ch\frac{z}{b} - \alpha ch\frac{z}{b}(1-c)\right]^{-2}\left\{(1-x)sh(1+x)\frac{z}{b} + \alpha(1-c-x)sh(1-c+x)\frac{z}{b} + \right.$$

$$\left. + (1-|x-c|)sh(1+|x-c|)\frac{z}{b} + \alpha(1-c-|x-c|)sh(1-c+|x-c|)z\right\}. \qquad (27)$$

We'll use below the following obvious inequalities

$$shkt \leq \exp((k-1)t)sht \quad (0 \leq k \leq 1), \; sht \geq \frac{1}{2}(1-\exp(-2t_1))\exp(t), \quad (t \geq t_1 > 0)\cdot$$

Hence

$$sh(1+x)\frac{z}{b} = sh\frac{1+x}{2}2\frac{z}{b} \leq \exp\left(-(1-x)\frac{z}{b}\right)sh2\frac{z}{b},$$

$$sh(1-c+x)\frac{z}{b} = sh\frac{1-c+x}{2}2\frac{z}{b} \leq \exp\left(-(1+c-x)\frac{z}{b}\right)sh2\frac{z}{b},$$

$$sh(1+|x-c|)\frac{z}{b} = sh\frac{1+|x-c|}{2}2\frac{z}{b} \leq \exp\left(-(1-|x-c|)\frac{z}{b}\right)sh2\frac{z}{b},$$

$$sh(1-c+|x-c|)\frac{z}{b} = sh\frac{1-c+|x-c|}{2}2\frac{z}{b} \leq \exp\left(-(1+c-|y-c|)\frac{z}{b}\right)sh2\frac{z}{b},$$

$$\left(ch\frac{z}{b} - \alpha ch(1-c)\frac{z}{b}\right)^{-2} \leq 4\left(1-\exp\left(-\frac{8}{3b}\right) - \alpha\exp\left(-\frac{4}{3c}\right)\right)^{-2}\exp\left(-2\frac{z}{b}\right).$$

Considering these inequalities in (27) we obtain

$$\left|\frac{\partial g}{\partial z}\right| \leq 2\left[1-\exp\left(-\frac{8}{3b}\right) - \alpha\exp\left(-\frac{4}{3c}\right)\right]^{-2}\exp\left(-2\frac{z}{b}\right)sh2\frac{z}{b}\left\{(1-x)\exp\left(-(1-x)\frac{z}{b}\right) + \right.$$

$$+ \alpha(1-c-x)\exp\left(-(1+c-x)\frac{z}{b}\right) + (1-|x-c|)\exp\left(-(1-|x-c|)\frac{z}{b}\right) +$$

$$\left. + \alpha(1-c-|x-c|)\exp\left(-(1+c-|x-c|)\frac{z}{b}\right)\right\}.$$

Considering $\exp\left(-2\frac{z}{b}\right)sh2\frac{z}{b} \leq \frac{1}{2}$, we obtain

$$\left|\frac{\partial g}{\partial z}\right| \leq \left[1-\exp\left(-\frac{8}{3b}\right) - \alpha\exp\left(-\frac{4}{3c}\right)\right]^{-2}\left\{(1-x)\exp\left(-(1-x)\frac{z}{b}\right) + \right.$$

$$+ \alpha(1+c-x)\exp\left(-(1+c-x)\frac{z}{b}\right) + (1-|x-c|)\exp\left(-(1-|x-c|)\frac{z}{b}\right) +$$

$$\left. + \alpha(1+c-|x-c|)\exp\left(-(1+c-|x-c|)\frac{z}{b}\right)\right\}.$$

This at $\frac{4}{3}n \le \frac{\beta_n}{h} \le z < n\pi, 0 \le x \le 1,\ \ 1 \le n \le \frac{1}{h}$ we obtain

$$\left|\frac{\partial g}{\partial z}\right| \le \left[1-\exp\left(-\frac{8}{3b}-\alpha\exp\frac{4}{3c}\right)\right]^{-2}\left\{(1-x)\exp\left(-(1-x)\frac{4}{3}n\right)+\right.$$

$$+\alpha(1-x+c)\exp\left(-(1+c-x)\frac{4}{3}n\right)+(1-|x-c|)\exp\left(-(1-|x-c|)\right)\frac{4}{3}n+$$

$$\left.+\alpha(1+c-|x-c|)\exp\left(-(1+c-|x-c|)\right)\frac{4}{3}n\right\}.$$

Then

$$\left|g\left(x,\frac{\beta_n}{h}\right)-g(x,n\pi)\right| \le \left[1-\exp\left(-\frac{8}{3b}\right)-\alpha\exp\left(-\frac{4}{3c}\right)\right]^{-2}\times$$

$$\times\left\{(1-x)\exp\left(-(1-x)\frac{4}{3}n\right)+\alpha(1+c-x)\exp\left(-(1+c-x)\frac{4}{3}n\right)+\right.$$

$$+(1-|x-c|)\exp\left(-(1-|x-c|)\frac{4}{3}n\right)+$$

$$\left.+\alpha(1+c-|x-c|)\exp\left(-(1+c-|x-c|)\frac{4}{3}n\right)\right\}\frac{(n\pi)^3}{6}h^2.$$

Estimate R_1: $R_1 \le L\frac{(1+\alpha)}{4}\pi^3h^2\left(1-\exp\left(-\frac{8}{3b}\right)-\alpha\exp\left(-\frac{4}{3c}\right)\right)^{-2}$,

where $L=\frac{4}{\pi^4}\max|\varphi'''(x)|$.

Thus $|u-u_h| \le L\left\{\frac{1}{3}+\frac{1+\alpha}{4}\pi^3\left(1-\exp\left(-\frac{8}{3b}\right)-\alpha\exp\left(-\frac{4}{3c}\right)\right)^{-2}\right\}h^2$.

4. CONCLUSIONS

In this paper for non-local mixed problem, the error of the discrete Fourier method is estimated effectively, i.e. the error is estimated with the aid of the known data, which are used in the process of intelligent modeling. This permits us to accurately estimate the error of the approximate solution and apply this method to the intellectual solution of the specified concrete problems considering the acquired (heuristic) contextual knowledge. Algorithms and software were developed to solve the problems posed by the discrete analogue of the Fourier method. The program was written in C++ language. For the various test samples based on the intellectual analysis of the source data. The software for the solution of the problems was developed and numerical experiments were introduced, confirming the reliability for the obtained results.

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