POSITION ADAPTATION OF CANDIDATE SOLUTIONS BASED ON THEIR SUCCESS HISTORY IN NATURE-INSPIRED ALGORITHMS

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Abstract: In this study a new technique for generating potential solutions in biology-inspired algorithms is proposed. Mentioned technique uses a historical memory of successful positions to guide them in different directions and thus to improve their exploration and exploitation abilities. The proposed method was applied to three bionic algorithms. Modified and original versions of these heuristics were evaluated on a set of various benchmark functions. It was established that the new technique allows finding better solutions with the same computational effort.

Key words: biology-inspired algorithms, firefly algorithm, bat algorithm, cuckoo search algorithm, position adaptation.

1. INTRODUCTION

In the last several years the development of heuristic optimization methods and their modifications for solving various types of problems attracts more attention from the computational intelligence community [1]. However, every search algorithm needs to address the exploration and exploitation of a search space. Exploration is the process of visiting entirely new regions of a search space, while exploitation is the process of visiting those regions of a search space within the neighbourhood of previously visited points [2]. A variety of ideas has been proposed to seek the balance between exploration and exploitation of biology-inspired and evolutionary algorithms, which includes parameter adaptation methods, population size control, island models and many others (for example, [3], [4]).

Still, sometimes ideas used in one class of algorithms are not known to the researchers of other class of algorithms. For example, the selection mechanisms used in genetic algorithms (GA) [5] are rarely applied in particle swarm optimization methods (PSO) [6] or differential evolution (DE) [7] algorithms used for real-parameter optimization [8]. Similarly, some ideas of DE, PSO or other bionic algorithms could be applied to alternative population-based optimizers.

One of the valuable ideas proposed for the DE algorithm in [9] is to use an archive of potentially good solutions, which is limited in size and updated as the search proceeds. This idea is similar to the external non-dominated set of solutions used in multi-objective optimizers such as SPEA or SPEA2 [10].

The advantage of the archive is that it contains promising solutions that appear to have valuable information about the search space and its promising regions, therefore indicating the history of algorithms' successful search [9]. The idea of using such information could be applied to any population-based method. In this paper the idea of applying the success-history based archive of potentially good solutions is tested for three biology-inspired algorithms, namely the Firefly Algorithm (FFA) [11], the Cuckoo Search Algorithm [12] and the Bat Algorithm [13].

The rest of the paper is organized as follows. Firstly, in the Section 2 a brief description of listed biology-inspired algorithms is given. Then in the Section 3 the proposed approach is introduced. The results obtained from the experiments are demonstrated in the Section 4. Finally, the Section 5 concludes the paper.

2. BIOLOGY-INSPIRED ALGORITHMS

The Firefly Algorithm (FFA) [11], the Cuckoo Search Algorithm (CSA) [12] and the Bat Algorithm (BA) [13] were used for the experiments. These biology-inspired algorithms are the population-based heuristics, which were chosen due to their high performance on various optimization problems. The basic idea of the listed algorithms involves generating the set of the potential solutions called individuals and moving them towards the global optima according to some rules established for a given algorithm.

Also all of the used heuristics lay the foundations for development of the various bionic algorithms. Thus, in this section brief description of the considered population-based algorithms for solving the optimization problems, to which the success-history based potential solutions' position adaptation was applied, are presented.

2.1. The Firefly Algorithm

Firefly algorithm (FFA) [11] is inspired by the flashing behaviour of fireflies. Each candidate solution called firefly is represented by its coordinates in the search space. Light intensity is associated with attractiveness of a firefly, and such attraction enables the fireflies with the ability to subdivide into small groups and each subgroup swarm around the local modes. This algorithm has been applied in continuous optimization, travelling salesman problem, clustering, image processing and feature selection.

FFA was developed by Xin-She Yang at Cambridge University in 2007; this algorithm uses the following three idealized rules [11]:

• all fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex;

• attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less bright one will move towards the brighter one; if there is no brighter firefly than a particular one, it will move randomly;

• the brightness of a firefly is affected or determined by the landscape of the objective function (for a maximization problem, the brightness can simply be proportional to the value of the objective function).

For simplicity the attractiveness of a firefly is determined by its brightness which in turn is associated with the encoded objective function.

2.2. The Cuckoo Search Algorithm

The Cuckoo Search Algorithm (CSA) is an optimization algorithm developed by Xin-She Yang and Suash Deb in 2009 [12]. It was inspired by the obligate brood parasitism of some cuckoo species by laying their eggs in the nests of other host birds (of other species). Similar to the Firefly Algorithm, the CSA heuristic also uses the idealized rules:

• each cuckoo lays one egg at a time, and dump its egg in randomly chosen nest;

• the best nests with high quality of eggs will carry over to the next generations;

• the number of available host nests is fixed, and the egg laid by a cuckoo is discovered by the host bird with a given probability (for simplicity, this last assumption can be approximated by the fraction of the nests which should be replaced by the new nests or in other words with the new random solutions).

For a maximization problem, the quality or fitness of a solution can simply be proportional to the value of the objective function. The following simple representations can be used: each egg in a nest represents a solution, and a cuckoo egg represents a new solution, the aim is to use the new and potentially better solutions (cuckoos) to replace a not-so-good solution in the nests.

When generating new solutions, a Levy flight [14] is performed: the random walk via Levy flight is more efficient in exploring the search space as its step length is much longer in the long run. The Levy flight essentially provides a random walk while a random step length is drawn from a Levy distribution, which has an infinite variance with an infinite mean. Some of the new solutions should be generated by Levy walk around the best solution obtained so far, this will speed up the local search. However, a substantial fraction of the new solutions should be generated by far field randomization and whose locations should be far enough from the current best solution, this will make sure the system will not be trapped in a local optimum.

2.3. The Bat Algorithm

Xin-She Yang proposed the Bat Algorithm (BA) in 2010 [12]: it was inspired by the research on the social behaviour of bats, namely the BA heuristic is based on their echolocation behaviour. Bats use a type of sonar (echolocation) to detect prey, avoid obstacles, and locate their roosting crevices in the dark. These bats emit a very loud sound pulse and listen for the echo that bounces back from the surrounding objects.

The Bat Algorithm is similar to the Particle Swarm Optimization (PSO) algorithm [6]: here each candidate solution called bat is represented by its coordinates in the search space and velocity. Bats fly randomly with some velocity at the new position with a fixed frequency, varying wavelength and loudness to search for prey (the global optima). They can automatically adjust the wavelength (or frequency) of their emitted pulses and adjust the rate of pulse emission, depending on the proximity of their target.

Furthermore, the loudness and the rate of pulse emission have to be updated accordingly as the iterations proceed. As the loudness usually decreases once a bat has found its prey, while the rate of pulse emission increases, the loudness can be chosen as any value of convenience. It is assumed that the loudness varies from a large (positive) number to a minimum constant value, which means that a bat has just found the prey and temporarily stop emitting any sound.

3. PROPOSED APPROACH

In this study, the success-history based potential solutions' position adaptation (SHPA) for improving the search diversity of biology-inspired algorithms is introduced. The key concept of the proposed technique can be described as follows.

First of all, the initial population for a given biology-inspired algorithm is generated. To be more specific, the set of potential solutions called individuals and represented as real-valued vectors with length D, where D is the number of dimensions for a given optimization problem, is randomly generated. Additionally, the external archive for best found positions is created. The size of this archive is chosen by the end-user and stays the same during the work of the bionic algorithm, but at the beginning it is empty.

For each individual the local best found position (the best found position by a given individual) in the search space is also saved. Thus, initially the local best for each individual is its current coordinates. If later the improved position will be discovered, then it will be used as the local best and the previous one will be stored in the external archive.

The process of the external archive update can be described with the following pseudo-code for a minimization problem:

```
A is the external archive
A_s is the external archive size
The individuals stored in the archive are Ai, where i =
1,…,A_s
The current number of individuals stored in the archive is
k
N is the population size
The individuals in the population are Pj, j = 1,…,N
The local best for each Pj is localj, j = 1,… ,N
The objective function is f
For each individual Pj (j = 1,…,N)
  If f(Pj) < f(localj)
        If (k + 1) ≤ A_s
              Ak+1 = localj
              k = k + 1
        End If
        If (k + 1) > A_s
              Randomly choose the integer r from 1 to A_s
              If f(localj) < f(Ar)
                    Ar = localj
              End If
        End If
        localj = Pj
  End If
End For
```

Fig. 1. Pseudo-code of the proposed technique for the minimization problem

Later when individuals change their position in the search space according to the formulas given for considered biology-inspired algorithm they can use the individuals stored in the archive with some probability. For example, let's consider the Bat Algorithm [12]. As was mentioned in the previous section each i-th individual from the population of the size N in the Bat Algorithm is represented by its coordinates $x_i = (x_{i1}, \ldots, x_{iD})$ and velocity $v_i = (v_{i1}, \ldots, v_{iD})$, where D is the number of dimensions of the search space. The following formulas are used for updating velocities and locations/solutions in the BA approach:

$$v_i(t+1) = v_i(t) + \left(x_i(t) - x^*\right) \cdot f_i; \qquad (1)$$

$$x_i(t+1) = x_i(t) + v_i(t+1); \qquad (2)$$

In these formulas t and $(t + 1)$ are the numbers, which indicate the current and the next iterations, x^* is the currently best found solution by whole population, and f_i

is the frequency of the emitted pulses for the *i*-th individual. Thus, with the probability p_a instead of the x^* the randomly chosen individual A_i from the external archive (if it is not empty) will be used. It is done with the expectation that individuals will move in multiple directions and, therefore, will be able to find better solutions.

For other two algorithms, FFA and CSA, the archive was used in the same way: with probability p_a the current point of attraction x^* was changed to stored solution A_i. To be more specific, in the CSA approach individuals were sorted according to the objective function. Then the worst ones were removed from population and new individuals instead of them were generated by using the external archive with a given probability p_a. On the other hand, in the FFA approach firefly or individual moves towards another firefly or individual if the latest has better objective function value [11]. While using the proposed technique for the FFA approach the firefly can be moved also towards individuals from the external archive.

The success-history based potential solutions' position adaptation strategy depends on the operators used by the population-based algorithm, probability p_a and archive size A_s. If the probability p_a is set to 0, then the standard version of the chosen biology-inspired algorithm is executed.

4. EXPERIMENTAL RESULTS

In this study the following 10 benchmark problems taken from [15] were used in experiments: the Rotated Discus Function, the Different Powers Function, the Rotated Rosenbrock's Function, Schwefel's Function, the Rotated Ackley's Function, the Rotated Griewank's Function, the Rotated Katsuura Function, the Rotated Lunacek bi-Rastrigin Function, the Rotated Weierstrass Function and Rastrigin's Function. These benchmark functions were considered to evaluate the robustness of the chosen biology-inspired algorithms with and without the proposed adaptation technique and to compare the obtained results.

As was mentioned before, for experiments the following three biology-inspired algorithms were used: the Firefly Algorithm, the Cuckoo Search Algorithm and the Bat Algorithm. For each test function and dimension 51 independent program runs were performed (this number was chosen due to the experimental settings given in [15]), and the dimensionality of the search space was set to $D = 10$ and $D = 30$. The population size was equal to 100 and each program run stopped if the number of function evaluations was equal to 10 000D. The maximum archive A_s size was set to 50, and the probability to use the external archive varied from $p_a = 0$ to $p_a = 0.75$ with step 0.05. In case if p_a was equal to 0, the external archive was not used.

In the first three tables the results, obtained by biology-inspired algorithms listed above, are presented. The following notations are used in these tables: "+" means that the algorithm with the certain probability p_a was better compared to the original algorithm (without external archive) according to the Mann-Whitney statistical test with significance level p = 0.01 [16], similarly, "–" means that algorithm with archive was worse, and "=" means that there was no significant difference between their results.

So, in these tables the first row indicates the probability values, while the first column contains the function number (from 1 to 10) and the dimensionality (10 and 30).

Table 1. Results for the FFA approach with p_a from [0.05; 0.75]

p_a	*1 – 10*	*1 – 30*	*2 – 10*	*2 – 30*	*3 – 10*	*3 – 30*	*4 – 10*	*4 – 30*	*5 – 10*	*5 – 30*	*6 – 10*	*6 – 30*	*7 – 10*	*7 – 30*	*8 – 10*	*8 – 30*	*9 – 10*	*9 – 30*	*10 – 10*	*10 – 30*	*Total +/–/=*
0.75	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	**20/0/0**
0.7	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	**20/0/0**
0.65	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	**20/0/0**
0.6	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	**20/0/0**
0.55	+	+	+	+	+	+	+	+	=	+	+	+	+	+	+	+	+	=	+	+	**18/0/2**
0.5	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	**20/0/0**
0.45	+	+	=	+	+	+	+	+	+	+	+	+	+	+	+	+	+	=	+	+	**18/0/2**
0.4	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	=	=	+	+	**18/0/2**
0.35	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	=	=	+	+	**18/0/2**
0.3	+	+	=	+	+	+	+	+	+	+	+	+	+	+	+	+	=	=	+	+	**17/0/3**
0.25	+	+	=	+	+	+	+	+	+	+	+	+	+	+	+	+	=	=	+	+	**17/0/3**
0.2	+	+	=	+	+	+	+	+	=	+	=	+	+	+	+	+	=	=	+	+	**15/0/5**
0.15	+	+	=	+	=	+	+	=	+	+	=	+	+	+	+	+	=	=	+	+	**14/0/6**
0.1	=	+	-	+	=	=	+	+	+	+	=	=	+	+	+	+	=	=	+	+	**12/1/7**
0.05	=	+	=	+	=	=	+	+	+	+	=	=	+	+	+	+	=	=	+	+	**12/0/8**

Table 2. Results for the CSA approach with p_a from [0.05; 0.75]

p_a	*1 – 10*	*1 – 30*	*2 – 10*	*2 – 30*	*3 – 10*	*3 – 30*	*4 – 10*	*4 – 30*	*5 – 10*	*5 – 30*	*6 – 10*	*6 – 30*	*7 – 10*	*7 – 30*	*8 – 10*	*8 – 30*	*9 – 10*	*9 – 30*	*10 – 10*	*10 – 30*	*Total +/–/=*
0.75	=	=	=	=	+	=	+	=	=	+	=	=	=	+	+	=	=	+	+	+	**8/0/12**
0.7	=	=	=	=	+	=	+	=	=	+	=	=	=	+	+	=	=	+	+	+	**8/0/12**
0.65	=	=	=	=	+	=	+	=	=	+	=	=	=	+	+	=	=	+	+	+	**8/0/12**
0.6	+	=	=	=	=	=	=	=	=	+	=	=	=	+	+	+	=	+	+	+	**8/0/12**

0.55	=	=	=	=	+	=	=	=	=	=	=	=	=	+	+	=	=	+	+	+	**6/0/14**
0.5	=	=	=	=	+	=	=	=	=	=	=	=	=	=	=	=	=	+	+	+	**4/0/16**
0.45	=	=	=	=	+	=	=	=	=	=	=	=	=	=	=	=	=	+	+	+	**4/0/16**
0.4	=	=	=	=	+	=	=	=	=	=	=	=	=	=	=	=	=	=	+	+	**3/0/17**
0.35	=	=	=	=	+	=	=	=	=	+	=	=	=	+	=	=	=	=	+	+	**5/0/15**
0.3	=	=	=	=	+	=	=	=	=	=	=	=	=	=	=	=	=	+	+	+	**4/0/16**
0.25	=	=	=	=	=	=	+	=	=	=	=	=	=	=	=	=	=	=	+	+	**3/0/17**
0.2	=	=	=	=	+	=	=	=	=	=	+	+	=	=	=	=	=	=	+	+	**5/0/15**
0.15	=	=	=	=	+	=	=	=	=	=	+	=	=	=	=	=	=	+	+	+	**5/0/15**
0.1	=	=	=	=	+	=	=	=	=	=	=	=	=	+	=	=	=	+	+	+	**5/0/15**
0.05	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	+	+	**2/0/18**

Table 3. Results for the BA approach with p_a from [0.05; 0.75]

p_a	*1 – 10*	*1 – 30*	*2 – 10*	*2 – 30*	*3 – 10*	*3 – 30*	*4 – 10*	*4 – 30*	*5 – 10*	*5 – 30*	*6 – 10*	*6 – 30*	*7 – 10*	*7 – 30*	*8 – 10*	*8 – 30*	*9 – 10*	*9 – 30*	*10 – 10*	*10 – 30*	*Total +/–/=*
0.75	=	=	=	=	=	=	=	=	=	=	=	=	=	=	=	+	=	+	=	=	**2/0/18**
0.7	=	=	=	=	+	=	=	=	=	+	=	=	=	+	+	+	-	+	=	=	**6/1/13**
0.65	=	=	=	=	=	=	=	=	=	=	=	=	+	+	=	+	-	+	+	=	**5/1/14**
0.6	=	+	=	=	+	=	=	=	=	+	-	=	+	=	=	+	=	+	+	=	**6/1/13**
0.55	=	=	=	=	+	=	=	=	=	+	=	=	+	+	=	+	=	+	=	=	**6/0/14**
0.5	=	=	=	=	+	=	=	=	=	=	=	=	=	=	+	+	=	+	=	=	**4/0/16**
0.45	=	=	=	=	+	=	=	=	=	+	=	=	=	+	+	+	=	+	=	=	**6/0/14**
0.4	=	=	=	=	=	=	=	=	=	=	=	=	=	+	=	+	=	+	+	=	**4/0/16**
0.35	+	+	=	=	+	=	=	=	=	=	=	=	=	+	=	+	=	+	+	=	**7/0/13**
0.3	=	+	=	=	+	=	=	=	=	+	=	=	=	=	=	+	=	+	+	=	**6/0/14**
0.25	=	+	=	=	=	=	=	=	=	+	=	=	=	+	=	+	=	+	=	=	**5/0/14**
0.2	=	+	=	=	=	=	=	=	=	+	=	=	=	=	=	+	=	=	+	=	**4/0/16**
0.15	=	+	=	=	+	=	=	=	=	+	=	=	=	+	=	+	=	+	+	=	**7/0/13**
0.1	=	+	=	=	=	=	=	=	=	+	=	=	=	+	+	+	=	+	=	=	**6/0/14**
0.05	=	=	=	=	+	=	=	=	=	+	=	=	=	+	=	+	=	=	=	=	**4/0/16**

Furthermore, the best version of the biology-inspired algorithms (without external archive or with it and the chosen probability p_a) was determined for each problem f and number of variables D according to the Friedman test [17]. Obtained results are presented in the fourth table. Each cell in this table contains the probability p_a of the best variant of bionic algorithm, and it is equal to 0 if the external archive wasn't used.

Table 4. The best variants of the bionic algorithms according to the Friedman test

f	**D**	***FFA***		***CSA***		***BA***	
		p_a	F_{min}	p_a	F_{min}	p_a	F_{min}
1	10	0.75	19274.2	0.6	69134.5	0.35	85792.5
	30	0.7	71258.1	0.2	152223	0.15	197266
2	10	0.65	2482.05	0.55	5442.97	0.05	7825.4
	30	0.75	21681.1	0.45	42918.8	0.35	45870.9
3	10	0.65	471.696	0.55	892.769	0.55	1009.31
	30	0.75	10744.5	0.1	18389.1	0.1	11151.3
4	10	0.65	1922.83	0.7	2307.52	0.05	1502.87
	30	0.5	8754.65	0.75	9061.45	0.5	8874.91
5	10	0.05	20.6548	0.05	20.4859	0.5	20.4421
	30	0.45	21.1703	0.75	21.1604	0.05	20.9942
6	10	0.75	739.841	0.2	1153.84	0.75	1517.97
	30	0.7	8051.63	0.75	11860.6	0.2	12001.4
7	10	0.15	2.2297	0.75	1.5748	0.35	1.1519
	30	0.6	4.2551	0.75	3.5816	0.05	2.7248
8	10	0.1	36.8734	0.75	39.537	0.1	34.4498
	30	0.5	198.484	0.6	203.549	0.5	171.09
9	10	0.7	11.4832	0.7	12.6263	0.05	11.3296
	30	0.7	47.5238	0.75	47.3934	0.25	41.5811
10	10	0.7	63.9692	0.75	101.801	0.2	181.373
	30	0.75	471.645	0.55	892.543	0.35	1286.28

Thus, it was concluded that the proposed approach improves biology-inspired algorithms' ability to find better solutions. For all test problems regardless of the chosen probability p_a the modified versions demonstrated better results than the original ones for all test functions with the same amount of the computational effort.

It was quite obvious that the success-history based potential solutions' position adaptation technique significantly increases the workability of the FFA approach: improvements were achieved for all test functions and dimensions according to the Mann-Whitney statistical test with significance level $p = 0.01$. Also the higher values of the probability p_a allow finding better solutions more frequently.

According to the obtained results, the Cuckoo Search Algorithm similar to the Firefly Algorithm demonstrates better results with probability pa greater than 0.55, while the Bat Algorithm shows statistically significant improvements by using the external archive with much smaller probability p_a (less than 0.5).

5. CONCLUSION

In this paper the success-history based potential solutions' position adaptation for bionic algorithms was proposed. This technique modifies the coordinates update procedure for biology-inspired algorithms by changing their attraction direction. Despite the simplicity of such approach and its little computational overhead, it allowed significant improvements for all three methods used in the experiments. Moreover, no performance loss was observed in any of the experiments, and for the FFA with p_a greater than 0.55 there were improvements for all the experiments.

The usage of the external archive of potential solutions, its update procedures and incorporation of stored solutions into the search procedure is an important topic of future research in the area of biology-inspired and evolutionary algorithms. Future research plans as well as ongoing research revolve around implementation of the proposed approach to the cooperative biology-inspired algorithms and expanding the concept of a heterogeneous algorithmic collective for solving data mining, decision support, control systems design and other problems of complex systems modelling and optimization.

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