DETERMINATION OF OPTIMAL COMPOSITION OF TEAM OF EXECUTORS FOR MULTISTAGE SERVICE SYSTEM

E. V. Bolnokina, S. A. Oleinikova, O. Ja. Kravets

Voronezh state technical univercity
e-mail: csit@bk.ru
Russian Federation

Abstract: In this paper, the object of the study is multistage systems, at the input of which comes a stream of requests that require executing a set of serial-parallel jobs for their support. The subject of the research is the optimization of the problem of assigning jobs to executors in such a system. As a result, a formalization of the problem is proposed, including a nonlinear objective function and recursive constraints.

Key words: organizational management, assignment problem, multistage servicing system, formalization, algorithm

1. INTRODUCTION

The multistage production systems, at the input of which comes a stream of requests requiring the execution of mutually dependent jobs, are investigated. It is assumed that the solution of any problem requires the implementation of a set of serial-parallel operations, each of which can be performed by a group of executors. Each executor has some preferences regarding the execution of job with other executors, which affects the quality of service. The subject of the research is the determination of such a composition of group of executors for executing each job, which allows to achieve optimal performance of the system.

The special cases of this problem have been thoroughly studied, and the corresponding solutions have been obtained. The problem of determining n executors for n jobs is a classic assignment problem. The Hungarian method of solving this problem is widely known. In addition, there is an approach to describe it by means of linear programming.

However, the studied problem has several specific features that determine the need to develop new approaches to solve it. This, in particular, is the requirement for the execution of job by several executors and the existence of mutual dependence between the jobs. For these reasons, the problem under study can no longer be described by means of linear programming. In this regard, the standard solution procedures should be upgraded considering the human factor. In addition, the existence of interdependence between jobs complicates the requirement to estimate the quality of work. This is because the immediately preceding operations will directly affect the quality of the job execution.

Thus, the formalization of the problem of choosing the optimal composition of executors and the development of an algorithm for its solution is relevant and practically important.

2. STATEMENT OF THE PROBLEM AND ITS SPECIFIC FEATURES

The problem of organizational management of the system, the distinctive feature of which is a set of mutually dependent jobs, is considered. Each job requires a specified number of executors for its execution, and each executor executes it with a certain average efficiency. In the general case, any executor can choose one of a set of actions, with the result that the job executed by him/her (together with his/her colleagues) turns out to be of some quality. It is assumed that the number of executors m is enough to execute all the jobs n. It is necessary to assign executors to jobs in such a way that the overall quality of the jobs executed (or the quality of the completed request) would be maximum.

Let us consider the specific features of this problem. There is a classical assignment problem, which solves the optimization problem of distributing n executors to n jobs, provided that each executor is given a certain job performance indicator. Compared with the classic case, the problem has several specific features. One of them is the joint performance of a certain operation by several executors. This feature should be considered for the following reasons. Firstly, the result of the execution of the job by a team of executors is a certain indicator that includes the contribution of each executor. Therefore, the effective distribution of such executors can increase this indicator, and vice versa. Secondly, teamwork always imposes some personal preferences. In the case of comfortable cooperation between the executors, their joint result will be better. The opposite is also true: with executors' negative attitude towards each other, it is extremely difficult to expect high quality work. In such a situation, as a rule, the performance of each individual executor decreases. Therefore, the consideration of the human factor is extremely important in solving the problem under study.

Another specific feature is related to the presence of interrelations between jobs. As noted in [1], the quality of one job is the basis for calculating the quality of the jobs directly related to it.

Thus, there are specific features that were not investigated in solving similar problems. They do not allow the use of existing approaches to solving the problem. In particular, the objective function and constraints will no longer be described by linear dependencies. In this regard, it is necessary to formalize the problem and develop an algorithm for its solution.

3. ANALYSIS OF EXISTING APPROACHES TO SOLVING THE STIMULATION PROBLEM

Consider a special case of solving this problem. Suppose there are n executors who need to be assigned to n jobs. Let the performance of the executor i be known when executing the job j. The result is the matrix of size n×n, which serves as an objective function.

Obviously, in this case there is n! different acceptable solutions. Existing methods are intended to narrow down the number of solutions to look through.

The classic assignment problem is a special case of the traffic problem (when suppliers' stocks and customers' requirements are equal to 1). To solve the traffic problem, there are such methods of solution as the potential method, the north-west corner method. Therefore, the traffic problem can also be solved by these methods. But the Hungarian method [2, 3] is more effective for solving this problem. Its essence is the successive transition from the original matrix to the matrix, which is equivalent to it and contains the so-called system of n independent zeros. This means that no two zeros belong to one row or one column. In other words, each row and each column of a new matrix must contain a single zero. The cell containing this zero determines the correspondence between the executors and the jobs (the row is the executor; the column is the job).

Since the objective function and constrains of this problem are linear, it can be written in the form of a linear programming problem. In particular, the formalization is obtained if we introduce the notations x_{ij} indicating whether the executor i will execute the job j (value 1) or not (value 0). As a result, we obtain:

$$\begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\cdot x_{ij} \to \max \\ \sum_{j=1}^{n} x_{ij} = 1, \forall i = 1,...,n \\ \sum_{i=1}^{n} x_{ij} = 1, \forall j = 1,...,n \\ x_{ij} \in \{0,1\} \end{cases} \quad (1)$$

The generalized assignment problem is also known. It is also the generalized knapsack problem. In this problem, the number of executors has the size that does not coincide with the size of the number of jobs. The cost matrix w_{ij} is specified. Each element of this matrix shows the costs of the executor i for executing the job j.

In addition, the income c_{ij} is given, which can be obtained when the executor i executes the job j. It is necessary to optimally distribute the jobs to the executors.

This problem is known to be NP-hard. In this connection, various attempts are made to construct approximate and heuristic algorithms for its solution [4, 5]. In particular, the greedy strategy of searching for the distribution of jobs to executors was proposed.

However, the problem under study, compared with the generalized assignment problem, is complicated by the fact that the quality of job execution can only be determined by knowing the actions performed by each executor and the quality of all previous jobs. Since this is impossible, it is necessary to develop proprietary technique, which allows us to propose an acceptable solution with varying precision.

4. PROBLEM FORMALIZATION

To formalize the problem, we examine in more detail the components that affect the quality of the execution of the separate job j. First, due to mutual dependence, this will be the quality of the jobs immediately preceding it. In addition, as noted above, the result of the work is influenced by each of the executors. Depending on the action chosen by the executor, we obtain a certain level of quality.

Let us describe the problem under study mathematically. As in the classical problem, we introduce the variables x_{ij} into consideration. They will be defined as follows:

$$x_{ij} = \begin{cases} 1, \text{ if i will execute the job j} \\ 0, \text{ otherwise} \end{cases} \qquad (2)$$

Let the “comfort” matrix also be given, each element k_{ij} of which shows the degree of comfort of the work of the executor i with the executor j.

It is also assumed that each executor can perform one of the L actions that will lead to execution of his/her job in this or that way. Suppose that the function H_j is formed for each job j. This function describes the result of the job j execution depending on the actions of the executors and the quality of previous jobs. Without loss of generality, let this value consist of two components:

- quality of all previous jobs;
- quality of execution of specified job.

Then the quality can be described by the formula:

$$H_j = \psi_j + RES\left(j, a_{i_1}, ..., a_{i_j}\right) \qquad (3)$$

Here, ψ_j is the overall quality of all the jobs preceding the job j; $RES(j,a_{i1},\dots,a_{ij})$ is the result obtained during the selection of the action a_{i1} by the executor i_1; the action a_{i2} by the executor i_2, etc. The form of the function ψ_j was defined in [1]:

$$\psi_j = \begin{cases} 0, \text{ j has no predecessors} \\ \sum_{k=1}^{m} H_k \cdot \alpha_k, \text{otherwise} \end{cases} \tag{4}$$

The coefficients α_k will indicate the degree of influence of the quality of the previous job k on the quality of the job j. They must meet the following conditions:

$$\begin{cases} 0 \le \alpha_k \le 1 \\ \sum_{k=1}^{m} \alpha_k = 1 \end{cases} \tag{5}$$

The RES() function depends on which executors and how they execute this job. Suppose, in the general case, there is some function f_j, which determines the contribution of each executor to the final result. In the simplest case, this may be the linear relationship:

$$f_j\left(W_j\right) = \sum_{i \in W_j} c_{ij} \tag{6}$$

Then the efficiency of the job j execution can be described by the following formula:

$$RES\left(j, a_{i_1}, ..., a_{i_j}\right) = f_j\left(W_j\right) \cdot k_{W_j} \tag{7}$$

Here, k_{Wj} is the coefficient showing the degree of comfort of the work of the group of executors $w_1, \ldots, w_K$, which constitute a certain set of W_j. The formula (7) shows that if the executors of the set W_j work comfortably with each other, their performance increases, and vice versa. In this case, the dependence on the degree of comfort of the executors is linear.

Thus, if the job j has no predecessors, then the function H will depend only on the quality of the joint execution of the job j. Otherwise, there is also a dependence on the quality of the execution of previous jobs.

Suppose that we can select the final job n, the quality of which will show the quality of the entire completed request. Otherwise, we can enter the fictitious final job n allowing us to determine the quality of the entire request (by the formula (3)).

Consider the specifics of the function (7), taking into account the team of co-workers who will perform this work. The problem can be formulated as follows. Suppose that the set of executors $w_1, \ldots, w_K$ is known. Preferences of each executor regarding the work with other participants are also known. It is necessary to estimate the overall coefficient of comfort of all executors' work with each other.

We can suggest the following option as one of the possible ones. Let it be necessary to determine the efficiency of interaction between executors belonging to the set $W = \{w_1, \ldots, w_K\}$. All the executors' preferences of working with each other are described by the matrix k_{ij} (the coefficient of work of the i-th executor with the j-th one). Then the comfort of the work of all executors included in the set W_j will be determined by the formula:

$$k_{W_j} = \frac{\sum_{i:w_i \in W_j} \sum_{l:w_l \in W_j} k_{il}}{2K} \quad (8)$$

Here, K is the number of executors included in the set W.

The dependencies (3), (5), (7) and (8) will form the basis of the mathematical description of the problem. The source data are:

- number of executors m;
- number of jobs n, m> n;
- number of executors $n_1, n_2, \ldots, n_n$, assigned to each job j, j = 1, ..., n;
- estimated performance of execution of the job j – c_{ij} by the executor i;
- matrix k, each element k_{ij} shows the degree of comfort of the work of the executor i with the executor j (in the general case, $k_{ij} \neq k_{ji}$);
- matrix α, each element α_{ij} of which shows the degree of influence of the job j on the job i;
- functions $f_j(W_j)$ showing the contributions of the executors included in the set W_j to the result of the job j.

It is required to assign executors to each job so as to maximize the overall performance of execution of all the jobs. This requires that the quality of all operations is not lower than the specified value H. Formally, the problem can be described as follows:

$$\begin{cases} \sum_{j=1}^{n} f_j\left(W_j\right) \cdot k_{W_j} \to \max \\ H_n \geq H \\ \sum_{j=1}^{n} x_{ij} = 1, \forall i = 1,...,m \\ \sum_{i=1}^{n} x_{ij} = n_i, \forall j = 1,...,n \\ x_{ij} \in \{0,1\} \end{cases} \quad (9)$$

Since the function H_n is recursive and the objective function is nonlinear in general, the system (9) cannot be considered as a linear programming problem. In this regard, we shall find an algorithm for its solution, taking into account the main features of the problem.

5. THE GENERALIZED ALGORITHM FOR SOLVING THE PROBLEM

Considering the recursive type of the function H_r, it is necessary to use a backtracking algorithm as the algorithm for solving the problem. This choice is due to the fact that the calculated quality of the jobs H_r for this distribution may not correspond to the declared value of H. In this case, it is necessary to abandon the solution obtained and offer another method of assignment. On this basis, the general algorithm for solving the problem has the following structure:

Stage 1. Offer the best solution and form the sets $W_{1,\dots,}W_n$.

Stage 2. Recursively estimate the quality of the jobs H_r for a given distribution.

Stage 3. If $H_r<H$

Then cancel the found solution and go to stage 1

Otherwise, show the answer.

Consider the main steps of the algorithm in more detail. We are interested mainly in the algorithm for finding the best (in terms of the efficacy of the result) partitioning of the set of executors into subsets for job execution. As a basis, we shall use a technique similar to the branch and bound method in the travelling salesman problem. Based on the matrix that can be obtained on the basis of the objective function, we choose the best (in terms of efficiency) jobs for each executor and the best executor for each job. Next, we need to go through all the candidates for compliance with "executor"-"job" and determine the penalty for refusing to assign the given job to the given executor. The cell with the maximum penalty will be the assignment at this step. Considering the information received, the following candidates are selected taking into account the adjustment of the objective function by the function (8).

6. CONCLUSION

The purpose of this paper was to formalize the problem of assigning specialists to jobs and finding the algorithm to solve it. As a result, we can draw the following conclusions.

1. The specificity of the studied problem is analyzed, as well as the approaches to the solution of classical specific assignment problems. As a result, it was concluded that it was necessary to develop the proprietary technique for its solution.
2. The formalization of the problem is proposed. As a result, it can be concluded that the obtained problem is no longer a linear programming problem.
3. A general approach to its solution is described.

Further work will be to detail the proposed algorithm. Of special interest will be the approach to the algorithmization of the first stage and the solution of the problem of selecting the sets $W_{1,\dots,}W_n$.

REFERENCES

[1] Bolnokina, E.V., S.A. Oleinikova. Formalization of the problem of choosing stimulating mechanisms in the problem of organizational management of a

multistage production system. *Control Systems and Information Technologies*. No **4** (74), 2018, pp. 26-29.

[2] Vatutin, E.I. et al. *Basics of discrete combinatorial optimization*. Argamak-Media, 2016.

[3] Taha, H. A. *Introduction into operations research*. Williams, 2005.

[4] Cohen, R., L. Katzir, D. Raz. An Efficient Approximation for the Generalized Assignment Problem. *Information Processing Letters*. Vol. 100 Issue **4**, 2006. pp. 162-166.

[5] Shmoys D.B., E. Tardos. An approximation algorithm for the generalized assignment problem. *Mathematical Programming*, **62(3)**, 1993. pp. 461-474.

Information about the authors:

Evgenija V. Bolnokina - Researcher, Voronezh State Technical

Svetlana A. Oleinikova – Doctor of Technical Science, Professor**,** Voronezh State Technical University, Russian Federation

Oleg Ja. Kravets – Doctor of Technical Science, Professor**,** Voronezh State Technical University, Russian Federation

Manuscript received on 12 February 2019