THE HEURISTIC ALGORITHM FOR SOLVING THE OPTIMIZATION PROBLEM OF THE START TIME OF JOBS DETERMINING FROM THE GENERALIZED RESOURCE CRITERION POINT OF VIEW

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Abstract: The work is devoted to the compilation of a schedule for multi-stage servicing systems from the point of view of a generalized resource criterion. The basis of this algorithm is the set of heuristics that allow making a decision on the performance the job at a given time or executing it later. As a result, an algorithm that determines the set of jobs that must be started at a given time was developed. The novelty lies in the use of heuristics, allowing to solve this problem from the point of view of the selected criterion.

Key words: heuristics, algorithm, multi-stage service systems, generalized resource criterion, time constraints.

1. INTRODUCTION

Modern production service systems are complex mechanisms operating under severe resource and time constraints – for example, production conveyor, patient care. In the case when it is necessary to organize the servicing of a large number of applications, take into account the specifics of the resources used to carry out each job, the quality of the schedule made plays an important role. An optimally designed schedule will not only make efficient use of the resources, but also significantly improve the performance of the system itself. In this regard, the task of forming schedules for multi-stage serving systems under the conditions of time and resource constraints is highly relevant.

This problem belongs to the project management task class [1, 2]. Today many problems in this area have already been solved, and scheduling algorithms have been obtained. The most studied should include planning tasks with the criterion of early completion. The existing methods are based on the CPM (Critical Path Method), which allows solving the problem without taking into account resource constraints [3, 4]. The next stage in the development of the theory of project management is the task of planning with constraints on resources. The problem is called resource constrained project scheduling problem (RCPSP) [5-8].

This article is devoted to the development the heuristics, allowing to implement an algorithm for determining the start time of jobs at a given moment, taking into account the generalized resource criterion and time constraints. The second part describes the specifics of the problem and proposes a general approach to its solution. Heuristics for the formation of the scheduling algorithm are presented in the third part. The fourth part contains the main conclusions.

2. THE SPECIFICS OF THE PROBLEM AND THE EXISTING METHODS FOR ITS SOLUTION

The task of determining the start time for multiphase systems with a generalized resource criterion was described in details in [9]. It consists in the following. There is a system, the service in which is a set of series-parallel jobs. Each job is characterized by the duration to make it complete it and the amount of the resources. It is required to make a schedule for the implementation of all the jobs. From a mathematical point of view, it is necessary to solve the optimization problem of finding the start time of each job in order to achieve the optimization of the selected criterion, when fulfilling the restrictions. In this case, consider the following limitations.

A). Restrictions on the interdependence of jobs (reflect the possibility of performing the next job only after all the jobs immediately preceding it have been completed). Mathematically, this limitation can be described as follows:

$$w_i.t_{beg} \geq w_k.t_{beg} + w_k.dur, \forall w_k \in Pr_i \quad (1)$$

Here w_i.beg - is the time to start job i, and w_i.dur - is the duration of this job. Pr_i - is a set of jobs immediately preceding the job i.

B). Restriction on the amount of the required resources. This restriction means that the start time of each job must be chosen in such a way that the volume of resources used to complete all the jobs at any given time would not exceed the amount available. This restriction should also take into account the current schedule for servicing applications received by the system earlier. Given the assumption that there are several types of resources needed to perform any job, it can be mathematically described as follows:

$$\sum_{w_m \in W_t} w_m.r_i + R_{usd\,i}(t) \leq R_i(t), i = 1,..., M. \quad (2)$$

Here, W_t is the set of all the jobs that will begin at time t; $w_m.r_i$ is the amount of resources of type i (i = 1, ..., M) required to complete the job w_m; $R_{usd\,i}(t)$- the amount of resources of type i, which is already used in the system for servicing applications received earlier; R_i - available resources of type i.

C). In addition, the task will be solved taking into account the time constraints. This restriction means that the time T_{i_fact} that required for service the entire application i must not exceed the specified standard term T_i. Mathematically, this limitation can be described using inequality:

$$T_{i\_fact} \le T_i \tag{3}$$

As a criterion a generalized resource criterion is selected. It means that the start time of each job must be determined in such a way as to minimize the difference between the actual amount of resources used for each type and some value specified by the decision maker. Taking into account several types of resources, we obtain a multi-criteria task, the criterion of which can be written in the form:

$$\begin{cases} \sum_t \left(\sum_{j:w_j \in W_t} w_j . r_1 + R_{usd\,1}(t) - f_1(t) \right)^2 \to \min \\ \dots \\ \sum_t \left(\sum_{j:w_j \in W_t} w_j . r_M + R_{usd\,M}(t) - f_M(t) \right)^2 \to \min \end{cases} \tag{4}$$

Here the function f_j (t) determines the most appropriate level of resources of type j that is used at the time t.

The rationale for this choice is given in [9]. Also in [9], a general algorithm for solving the problem is presented. It is based on the sequential formation of the graph on each time interval, taking into account the constraints (1) and (2) and the objective function (4). At the end of the planning process, a time limit violation is checked (3). If condition (3) is fulfilled, then the planning process is completed, otherwise the time interval for return is selected, the solution found on it is prohibited and the schedule is re-formed.

Schematically, the generalized algorithm can be represented as follows (Figure1).

The general idea of the algorithm is as follows. At each stage i, the formation of a graph according to criterion (4) and resource constraints (2) is carried out. Since time constraints can be checked only at the end of the planning process, at this stage it is possible to obtain only a lower estimate of this quantity. If it exceeds a critical time, it means violation of time constrains and, as a consequence necessity to return to one of the previous stages. Starting from this stage it is required to make a schedule of jobs anew.

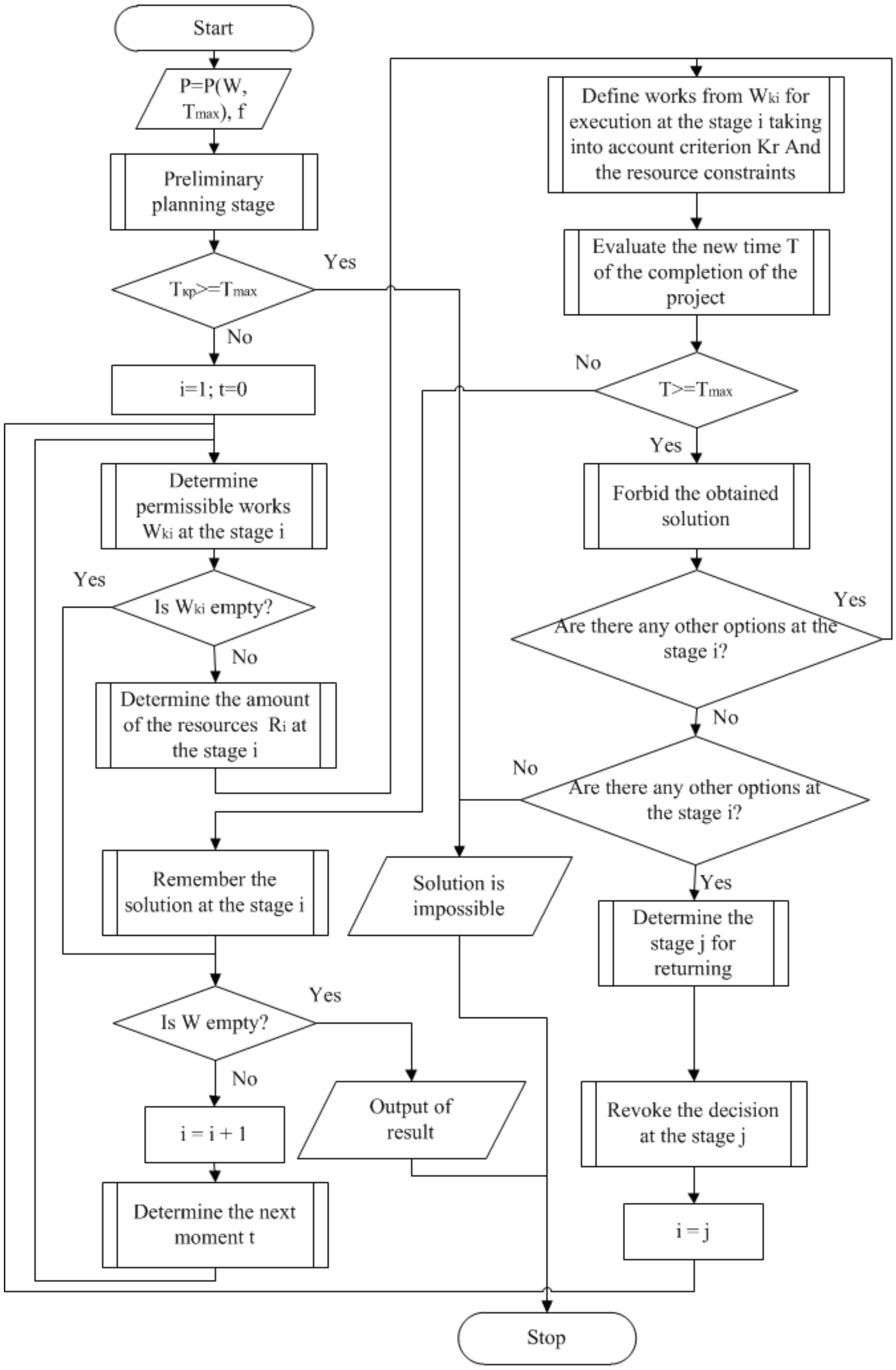

Figure 1. Generalized algorithm for creating a service schedule

3. HEURISTICS FOR SCHEDULING AT A GIVEN TIME INTERVAL

The greatest interest in the algorithm shown in figure 1, represent the following steps:

- determining the set of jobs W_{ki} for execution at the stage i;
- evaluation of the project completion time;
- selection the stage for the return (in the case of violation of time constraints).

To find the set of jobs that need to start in stage i, heuristics, taking into account the multiple criterion (4) and resource constraints (2), were developed. The general idea of the algorithm is based on the consistent assignment of the given time to jobs, based on their priority (which is determined on the basis of the temporary work reserve). As soon as the amount of resources used reaches a certain level determined by criterion (4), there will be a need to assess the appropriateness of assigning the remaining work at a given time, as long as the resource constraints (2) allow it. Heuristics, which will be the basis of the algorithm that determines the start time of jobs, are given below.

Heuristic 1 (H1). Suppose that at this stage in an arbitrary center the choice of the assignment of the work j is decided. Let the total amount of resources used at a given moment t in this center be determined by the vector $R_{usd}(t)=(R_{usd\,1}(t),\ldots, R_{usd\,m}(t))$. Suppose that the following inequalities hold:

$$\begin{cases} R_{usd\,1}(t)+w_j..r_1 \le f_1(t); \\ \ldots \\ R_{usd\,m}(t)+w_j.r_m \le f_m(t). \end{cases} \tag{5}$$

They mean that the level of resources currently in use, together with the resources needed to perform the job j, has not reached the value of f. In this case, the job must be done at this moment.

Heuristic 2 (H2). Suppose that at least one of the inequalities (5) is violated at this stage t_0. The job j must be done at a given time in the event that the inequality holds:

$$\begin{aligned} &\sum_{i=1}^{m}\left(R_{usd\,i}(j^+)-f_i(t_0)\right)^2+\sum_{i=1}^{m}\left(V_i(j^-)-\sum_{t>t_0}\left(f_i(t)-R^-_{usd\,i}(t)\right)\cdot h_i\right)^2< \\ &<\sum_{i=1}^{m}\left(R_{usd\,i}(j^-)-f_i(t_0)\right)^2+\sum_{i=1}^{m}\left(V_i(j^+)-\sum_{t>t_0}\left(f_i(t)-R^+_{usd\,i}(t)\right)\cdot h_i\right)^2. \end{aligned} \tag{6}$$

where $R_{usd\,i}(j^+)$ – the amount of resources of type i, taking into account the performance of the job j at this stage;

$R_{usd\,i}(j^-)$ – the amount of resources used of type i without taking into account the performance of the job j at this stage;

$V_i(j^-)$ – the total amount of type i resources necessary to perform all unplanned jobs in all remaining stages, provided that the job j will be performed later;

$V_i(j^+)$ – the total amount of type i resources necessary to perform all unplanned jobs in all remaining stages, provided that the job j is completed at this moment.

Formula for calculating the values of $V_i(j^-)$:

$$V_i\left(j^-\right) = \sum_{k \in W_{unp}} \left(w_k..r_i \cdot w_k.dlit\right) \tag{7}$$

W_{unp} - a lot of unplanned jobs (without taking into account the job of j).
The formula for $V_i(j^+)$ is similar and differs only in the composition of the set W_{unp}, which in this case will also contain the job of j.

A graphic illustration of this heuristic is shown in Figure 2.

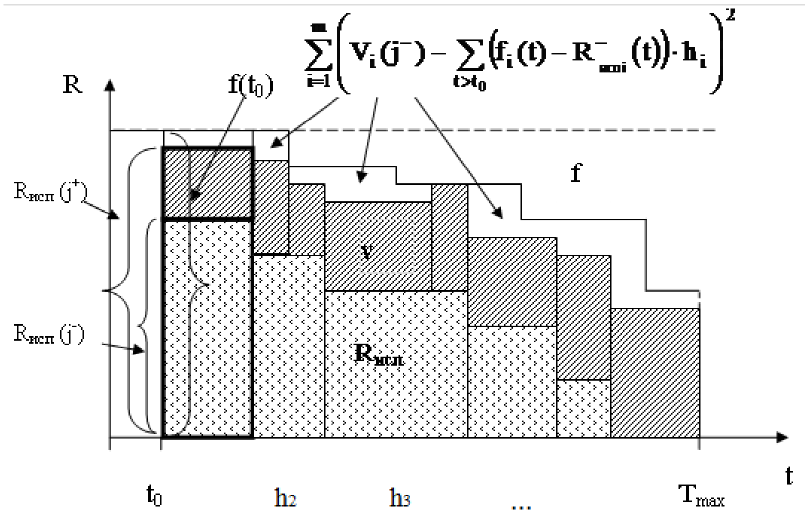

Figure 2. Illustration of heuristic 2

Let's explain this heuristic. The first term on the left and right sides of inequality (6) is a sum, each term of which makes it possible to estimate the degree of proximity of the current volume of resources of a given type to the corresponding objective function (taking into account the performance of the job j at a given time and taking into account its transfer, respectively). The next - is the degree of proximity of the amount of resources of each type used to perform of all unplanned jobs to the desired value. In this regard, the sum of these two terms will provide an approximate estimate of the objective function in the event that the controversial jobs are carried out at a given time or moved to a later date. The next heuristic gives an unambiguous answer to the necessity for planning this work in conditions of possible non-fulfillment of time constraints.

Heuristic 3 (H3). Suppose that a decision of the necessity of performing the highest priority job j from the set W on a given time interval is taken. The job j must be done at this time if at least one of the following restrictions occurs:

$$V_i(j^+) > \sum_{t>t_0} \left(f_i(t) - R^+_{usd\,i}(t)\right) \cdot h_i, \ i=1,\ldots,m. \tag{8}$$

Inequality (8) suggests that there is not enough free resources (taking into account the available schedule) of some type i for the system to perform all the remaining workloads within the given timeframe.

For criteria such as early completion with resource constraints, it makes no sense to disrupt the workflow for planning. This problem may be options when planning for the extraordinary work can be effective. The following heuristics are introduced due to the specifics of the optimality criterion.

Heuristic 4 (H4). Suppose that at a given moment t_i from the set of jobs W two subsets W_1 and W_2 are formed. W_1 includes the jobs that will be performed at a given time, and W_2 – the jobs that needs to be postponed to a later period. Assume that, based on the specifics of the set W_1 and the existing schedule, the next planning moment t_{i+1} is found. Then, if there is a job k in W_2 that does not degrade the value of the objective function and whose duration is less than the difference t_{i+1}-t_i, then it is expedient to assign the time t_i to the job k.

Heuristic 5 (H5). Suppose that from the point of view of all heuristics, it was decided that it would be expedient to transfer some job to the next time interval. Suppose that a new start time of this job was found. If in this case the job reserve has become negative, then the job must be performed at a given time. If resource constraints do not allow this, violations of time constraints and the necessity to return to earlier planning stages are fixed.

Heuristic 6 (H6). (assessment of violation of time constraints). If at assignment of the job of the given moment of time, at least one of the inequalities is fulfilled:

$$V_i(j^-) > \sum_{t>t_0} \left(f_i(t) - R^+_{usd\,i}(t)\right) \cdot h_i, \qquad i=1,\ldots,m, \tag{9}$$

where $V_i(j^-)$ defined by the (7), then this means that the time limits will be violated. In fact, this means that the remaining amount of resources, calculated on the basis of the available time, is not enough to fulfill of all the remaining amount of jobs. Heuristic data form the basis of an algorithm that allows to schedule on a given time interval. The scheme of the algorithm is shown in Figure 3.

The following notation is used in the figure: C1 is a sequential check of the conditions for the execution of the heuristics H1, H3 and H2, respectively; C2 - fulfillment of the conditions of the heuristics H5 or H6. The definition of the add vector is described in detail in [9], and the definition of the stage for return is described in [10].

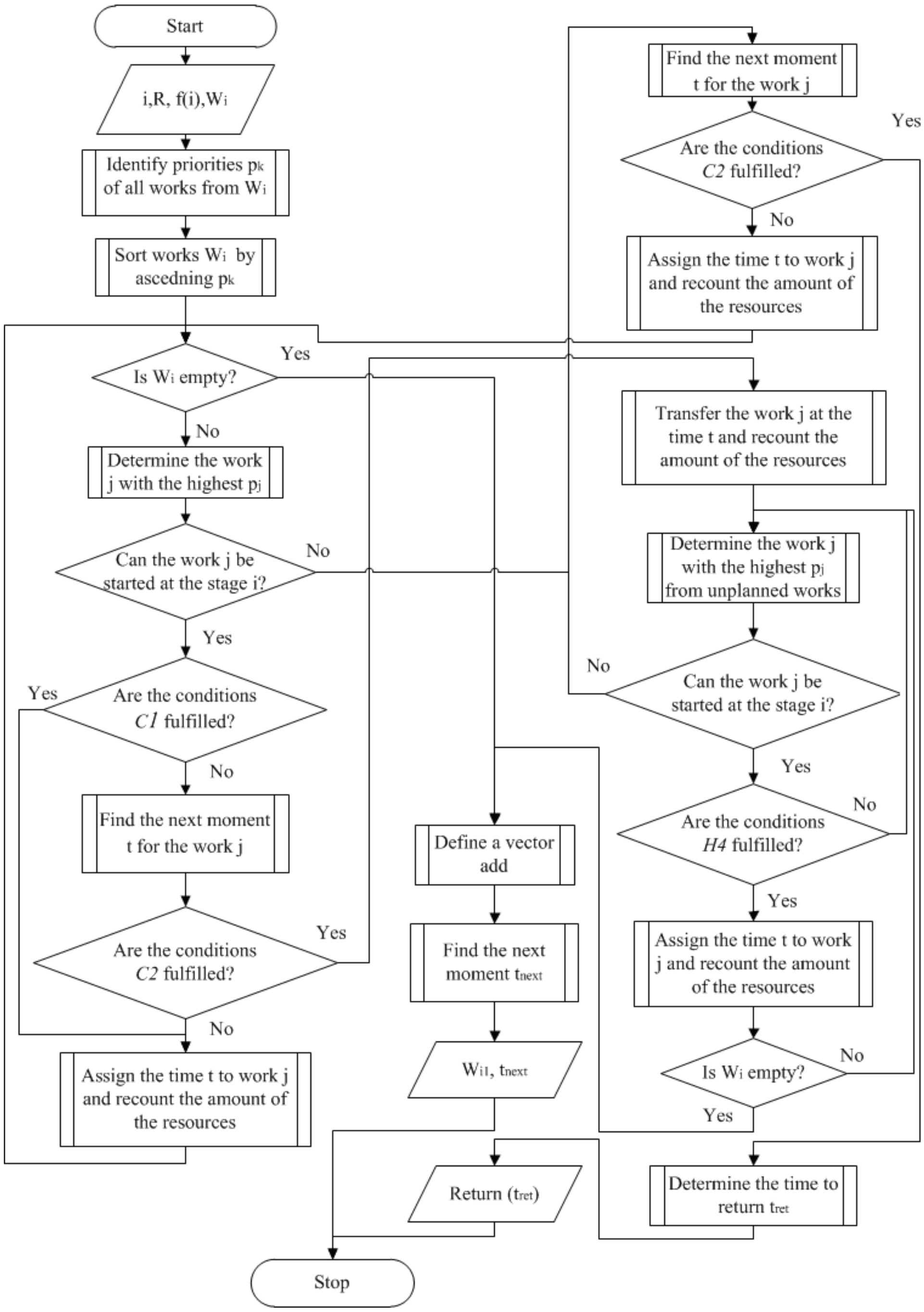

Figure 3. The scheme of the algorithm for determine of the set of jobs beginning at the interval i

4. CONCLUSIONS

The purpose of this work was to develop an algorithm based on heuristics and allowing to make the schedule at a given time interval in terms of criteria (4) and constraints (1) - (3). Analysis of the specifics of the problem and the existing approaches to its solution led to the following conclusions:

1. The feature of time constraints requires the use of an algorithm with a return, which is reflected in Fig. 1. A refund is carried out if, at the end of planning, the restriction on the execution time of the request is violated.
2. The specificity of the criteria used does not allow the use of existing approaches, in particular, the classical approach to solving the RCPSP problem.
3. In this regard, heuristics allowing to decide on the appointment of some job at a given time or transfer it to the next stage were developed.
4. The developed heuristics formed the basis of the algorithm for making the jobs execution schedule at a given time interval from the point of view of the generalized resource criterion.

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