THE MODELS OF THERMOELASTIC DEFORMATION OF THIN ELLIPTIC BOUNDARY CONTOURS PLATES WITH ACCOUNTING UNCERTAINTY FACTORS

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Abstract: A fuzzy-set technique is proposed for analyzing the uncertainty of the thermoelastic deformations characteristics of thin plates with external or internal boundary elliptical contours, taking into account the scatter errors of the values of the initial physical-mechanical and geometric parameters based on the use of the heuristic principle of generalization. The resulting analysis of the model estimation allow to draw conclusions about the reliable ranges of deviations in the values of the analyzed indicators of the intensity of the bending moments and stress concentration at various points of the contour of the hole when the dispersion of physical and mechanical parameters. Significant relationships of low-variant versions of the model are created for the base component, which is a homogeneous orthorhombic area of nonzero thickness and nonzero area in given coordinates. Axes passing through the middle of the element are defined. Half shafts define an elliptical contour. Stresses in the element are generated by the heat flux, which is symmetric and axial with respect to the coordinate system in the heat flux part.

Key words: fuzzy-set technique, thermoelastic deformations characteristics, thin plates, boundary elliptical contours, modeling, bending moments.

1. INTRODUCTION

Structural elements in the form of thin plates of circular and elliptical shape, plates with small holes of elliptical and circular shape are common elements of aerospace vehicles and industrial equipment.

The most relevant models of the stress state of such structures include models of their temperature deformation [1-5]. In particular, these are models of bending deformation of solid plates of elliptical shape caused by temperature differences on opposite flat faces; models of the thickness-symmetric thermally stressed state of plates with holes.

If the initial information about errors in the scattering of exogenous parameters in models of bending or flat temperature deformation of thin plates is correct statistical information, used to account for the effects of dispersion [6]. However, in most cases, there are no extensive correct statistical data on the spread that should be taken into account. In this regard, it may be recommended to describe the indeterminate endogenous parameters, using fuzzy set theory methods [6-11].

2. BASIC RELATIONS OF DETERMINISTIC VERSIONS OF THE MODELS UNDER STUDY

The basic relations of the deterministic version of this model are formulated for a structural element in the form of a single-connected anisotropic plate of orthorhombic class with a thickness h, that occupies an area $V=\{(x_1,x_2)\in S, -h/2\le x_3\le h/2\}$ in coordinate space $Ox_1x_2x_3$. The boundary elliptical contour L of the middle plane of the plate S has semi-axes a, b. The bending of the plate is caused by the action of a stationary temperature field that changes linearly along its thickness

$$T(x_1,x_2,x_3)=x_3\cdot\tau_0(x_1,x_2) \tag{1}$$

and there are no external forces on flat faces $x_3=\pm h/2$. The case of a rigidly fixed side surface of the plate is considered, and the condition is met on the side boundary surface $\Gamma=\{(x_1,x_2)\in L,\ x_3\in[-h/2,\ h/2]\}$:

$$(\tau_0(x_1,x_2))_\Gamma = T_0\,. \tag{2}$$

In this problem, the averaged characteristics of the stress state obtained using the methods of the theory of functions of generalized complex variables [5] – bending moments M_1, M_2, moment H_{12} and transverse forces:

$$M_1=-\gamma_1T_0, M_2=-\gamma_2T_0, H_{12}=N_1=N_2=0, \tag{3}$$

$$\begin{gathered}\gamma_1=D_{11}\alpha_1+D_{12}\alpha_2, \gamma_2=D_{12}\alpha_1+D_{22}\alpha_2,\\ D_{11}=E_1h^3/(12(1-\nu_1\nu_2)), D_{22}=E_2h^3/(12(1-\nu_1\nu_2)),\\ D_{12}=\nu_1E_1h^3/(12(1-\nu_1\nu_2))=\nu_2E_2h^3/(12(1-\nu_1\nu_2));\end{gathered} \tag{4}$$

E_1, E_2, ν_1, ν_2 – Young modules and Poisson coefficients of the plate material; α_1, α_2 – coefficients of thermal expansion of the plate material along elastic-equivalent directions. Thus, in accordance with the relations (3), (4), the endogenous characteristics of bending moments M_1, M_2 to be analyzed are described by functional dependencies

$$M_j=F_{M_j}(\,h, E_1,E_2,\nu_1,\nu_2,T_0,\alpha_1,\alpha_2) \tag{5}$$

The generalized plane stress state of the plate is caused by the action of a thickness-symmetric heat flow with averaged intensity q directed at an angle β to the coordinate direction Ox_1, which corresponds to the distribution of the average temperature function $T(x_1, x_2)$ in the middle plane of the plate

$$T(x_1, x_2) = q(x_1 \cos\beta + x_2 \sin\beta)\,. \tag{6}$$

According to [4], the stress at the point with the angular coordinate θ on the contour of the hole with a parametric description:

$$x_1 = a\cos\theta\,,\ x_2 = b\sin\theta \tag{7}$$

$$\begin{gathered}\sigma_\theta = F_{\sigma_\theta}(\alpha_t, q, a, b, E, \beta, \theta) = \\ = \alpha_t qER[(1-k^2)\cos(\theta+\beta) - (1+k^2)\cos(\theta-\beta)]\cdot[1-\cos 2\theta + k^2(1+\cos 2\theta)]^{-1}\end{gathered} \tag{8}$$

In expression (8) $k = b/a$; $R = (a+b)/2$; α_t – temperature coefficient of linear expansion of the plate material; E – Young modulus of the plate material.

3. GETTING FUZZY-SET REPRESENTATIONS OF THE ENDOGENOUS PARAMETERS OF THE MODELS

The fuzzy-interval descriptions of the form $a, b, h, E_1, E_2, \nu_1, \nu_2, G, T_0, \alpha_1, \alpha_2$ are introduced for exogenous parameters $\tilde{a}, \tilde{b}, \tilde{h}, \tilde{E}_1, \tilde{E}_2, \tilde{\nu}_1, \tilde{\nu}_2, \tilde{G}, \tilde{T}_0, \tilde{\alpha}_1, \tilde{\alpha}_2$

$$\begin{gathered}\tilde{a} = (a_1, a_2, a_3, a_4), \tilde{b} = (b_1, b_2, b_3, b_4), \tilde{h} = (h_1, h_2, h_3, h_4), \\ \tilde{E}_1 = (E_{11}, E_{12}, E_{13}, E_{14}), \\ \tilde{E}_2 = (E_{21}, E_{22}, E_{23}, E_{24}),\ \tilde{\nu}_1 = (\nu_{11}, \nu_{12}, \nu_{13}, \nu_{14}), \\ \tilde{\nu}_2 = (\nu_{21}, \nu_{22}, \nu_{23}, \nu_{24}), G = (G_1, G_2, G_3, G_4), \\ \tilde{T}_0 = (T_{01}, T_{02}, T_{03}, T_{04}),\ \tilde{\alpha}_1 = (\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}), \\ \tilde{\alpha}_2 = (\alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}).\end{gathered} \tag{9}$$

Using representations (9) и and fuzzy interval arithmetic rules [14-15]:

$$\begin{gathered}\tilde{D}_{11} = (E_{11}h_1^3/(12(1-\nu_{11}\nu_{21}), E_{12}h_2^3/(12(1-\nu_{12}\nu_{22}), \\ E_{13}h_3^3/(12(1-\nu_{13}\nu_{23}), E_{14}h_4^3/(12(1-\nu_{14}\nu_{24})), \\ \tilde{D}_{22} = (E_{21}h_1^3/(12(1-\nu_{11}\nu_{21}), E_{22}h_2^3/(12(1-\nu_{12}\nu_{22}), \\ E_{23}h_3^3/(12(1-\nu_{13}\nu_{23}), E_{24}h_4^3/(12(1-\nu_{14}\nu_{24})), \\ \tilde{D}_{12} = (\nu_{11}E_{11}h_1^3/(12(1-\nu_{11}\nu_{21}), \nu_{12}E_{12}h_2^3/(12(1-\nu_{12}\nu_{22}), \\ \nu_{13}E_{13}h_3^3/(12(1-\nu_{13}\nu_{23}), \nu_{14}E_{14}h_4^3/(12(1-\nu_{14}\nu_{24})), \\ \tilde{\gamma}_1 = ((\alpha_{11}+\nu_{11}\alpha_{21})E_{11}h_1^3\,/\,(12(1-\nu_{11}\nu_{21})), \\ (\alpha_{12}+\nu_{12}\alpha_{22})E_{12}h_2^3\,/\,(12(1-\nu_{12}\nu_{22})),\end{gathered} \tag{10}$$

$$(\alpha_{13}+\nu_{13}\alpha_{23})\ E_{13}h_3^3\,/\,(12(1-\nu_{13}\nu_{23})),$$
$$(\alpha_{14}+\nu_{14}\alpha_{24})E_{14}h_4^3\,/\,(12(1-\nu_{14}\nu_{24}))),$$
$$\tilde{\gamma}_2=((\alpha_{21}+\nu_{21}\alpha_{11})E_{21}h_1^3\,/\,(12(1-\nu_{11}\nu_{21})),$$
$$(\alpha_{22}+\nu_{22}\alpha_{12})E_{22}h_2^3\,/\,(12(1-\nu_{12}\nu_{22})),$$
$$(\alpha_{23}+\nu_{23}\alpha_{13})\ E_{23}h_3^3\,/\,(12(1-\nu_{13}\nu_{23})),$$
$$(\alpha_{24}+\nu_{24}\alpha_{14})E_{24}h_4^3\,/\,(12(1-\nu_{14}\nu_{24}))),$$
$$\tilde{M}_1=(-T_{01}(\alpha_{11}+\nu_{11}\alpha_{21})E_{11}h_1^3\,/\,(12(1-\nu_{11}\nu_{21})),$$
$$-T_{02}(\alpha_{12}+\nu_{12}\alpha_{22})E_{12}h_2^3\,/\,(12(1-\nu_{12}\nu_{22})),$$
$$-T_{03}(\alpha_{13}+\nu_{13}\alpha_{23})\ E_{13}h_3^3\,/\,(12(1-\nu_{13}\nu_{23})),$$
$$-T_{04}(\alpha_{14}+\nu_{14}\alpha_{24})E_{14}h_4^3\,/\,(12(1-\nu_{14}\nu_{24}))),$$
$$\tilde{M}_2=(-T_{01}(\alpha_{21}+\nu_{21}\alpha_{11})E_{21}h_1^3\,/\,(12(1-\nu_{11}\nu_{21})),$$
$$-T_{02}(\alpha_{22}+\nu_{22}\alpha_{12})E_{22}h_2^3\,/\,(12(1-\nu_{12}\nu_{22})),$$
$$-T_{03}(\alpha_{23}+\nu_{23}\alpha_{13})\ E_{23}h_3^3\,/\,(12(1-\nu_{13}\nu_{23})),$$
$$-T_{04}(\alpha_{24}+\nu_{24}\alpha_{14})E_{24}h_4^3\,/\,(12(1-\nu_{14}\nu_{24}))).$$

In particular, for a plate made of a composite fiber material-fiberglass [5], when specifying fuzzy-interval initial parameters in the form:

$$\begin{gathered}\tilde{a}=(194l_*,198l_*,\,201l_*,\,203l_*),\\ \tilde{b}=(94l_*,97l_*,101l_*,102l_*),\ \ \tilde{h}=(3.8l_*,\,4l_*,\,4.2l_*,\,4.6l_*),\\ \tilde{E}_1=(3.78E_*,3.81E_*,3.83E_*,3.91E_*),\ \ \tilde{E}_2=(0.96E_*,1.0E_*,1.05E_*,1.12E_*),\\ \tilde{\nu}_1=(0.071,0.072,0.074,0.076),\ \ \tilde{\nu}_2=(0.26,\,0.28,\,0.29,\,0.30),\\ \tilde{G}=(0.378E_*,\,0.383E_*,\,0.385E_*,\,0.392E_*),\ \ \tilde{T}_0=(96,115,121,135),\\ \tilde{\alpha}_1=(0.66\alpha_*,\,0.69\alpha_*,\,0.71\alpha_*,\,0.74\alpha_*),\\ \tilde{\alpha}_2=(3.75\alpha_*,\,3.80\alpha_*,\,3.82\alpha_*,\,3.94\alpha_*),\\ l_*=10^{-3}[\text{м}],\ E_*=10^9[\text{Pa}],\end{gathered}\tag{11}$$

for exogenous parameters $\alpha, q, a, b, E, \beta$ in the ratio (8) fuzzy-interval descriptions of the type are introduced $\tilde{\alpha}, \tilde{q}, \tilde{a}, \tilde{b}, \tilde{E}, \tilde{\beta}$

$$\begin{gathered}\tilde{\alpha}_t=(\alpha_{t1},\alpha_{t2},\alpha_{t3},\alpha_{t4}),\tilde{q}=(q_1,q_2,q_3,q_4),\\ \tilde{a}=(a_1,a_2,a_3,a_4),\tilde{b}=(b_1,b_2,b_3,b_4),\\ \tilde{E}=(E_{*1},E_{*2},E_{*3},E_{*4}),\ \tilde{\beta}=(\beta_1,\beta_2,\beta_3,\beta_4),\end{gathered}\tag{12}$$

and corresponding representations in the form of superpositions α – slices

$$\tilde{\alpha}_t = \bigcup_{\alpha\in[0,1]} [\underline{\alpha}_{t\alpha}, \overline{\alpha}_{t\alpha}]\, \underline{\alpha}_{t\alpha} = (1-\alpha)\alpha_{t1} + \alpha\alpha_{t2}, \overline{\alpha}_{t\alpha} = \alpha\alpha_{t3} + (1-\alpha)\alpha_{t4};$$

$$\tilde{q} = \bigcup_{\alpha\in[0,1]} [\underline{q}_{\alpha}, \overline{q}_{\alpha}]\ \underline{q}_{\alpha} = (1-\alpha)q_1 + \alpha q_2,\ \overline{q}_{\alpha} = \alpha q_3 + (1-\alpha)q_4;$$

$$\tilde{a} = \bigcup_{\alpha\in[0,1]} [\underline{a}_{\alpha}, \overline{a}_{\alpha}]\ \underline{a}_{\alpha} = (1-\alpha)a_1 + \alpha a_2,\ \overline{a}_{\alpha} = \alpha a_3 + (1-\alpha)a_4; \quad (13)$$

$$\tilde{b} = \bigcup_{\alpha\in[0,1]} [\underline{b}_{\alpha}, \overline{b}_{\alpha}]\ \underline{b}_{\alpha} = (1-\alpha)b_1 + \alpha b_2,\ \overline{b}_{\alpha} = \alpha b_3 + (1-\alpha)b_4;$$

$$\tilde{E} = \bigcup_{\alpha\in[0,1]} [\underline{E}_{\alpha}, \overline{E}_{\alpha}]\ \underline{E}_{\alpha} = (1-\alpha)E_{*1} + \alpha E_{*2},\ \overline{E}_{\alpha} = \alpha E_{*3} + (1-\alpha)E_{*4};$$

$$\tilde{\beta} = \bigcup_{\alpha\in[0,1]} [\underline{\beta}_{\alpha}, \overline{\beta}_{\alpha}]\ \underline{\beta}_{\alpha} = (1-\alpha)\beta_1 + \alpha\beta_2,\ \overline{\beta}_{\alpha} = \alpha\beta_3 + (1-\alpha)\beta_4.$$

4. CONCLUSION

The resulting analysis of the model estimation allow to draw conclusions about the reliable ranges of deviations in the values of the analyzed indicators of the intensity of the bending moments and stress concentration at various points of the contour of the hole when the dispersion of physical and mechanical parameters.

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