QUALITY ASSESSMENT IN A MULTI-LEVEL CONTROL SYSTEM WITH DISCRETE RESPONSE FUNCTIONS

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Abstract: The article extends the authors' previous research to the case of discrete response functions. A process consisting of sequential operations is considered. For each operation, you can determine the currently available probability of its error-free (high-quality) execution. By the qualitative performance of the operation, we will understand the achievement of a given level of quality in quantitative proportion to the overall level. The purpose of the article is to build a quality management strategy in which a set of probabilities of quality levels not less than a certain threshold value is achieved, with minimal costs for carrying out appropriate measures. The corresponding optimization problem is presented and a heuristic multistep algorithm for its solution is given.

Key words: sequential operations, high-quality execution, quality management strategy, threshold value, classification of activities, discrete response functions.

1. INTRODUCTION

In [1], the concept of probabilistic control is detailed and its features for continuous response functions are considered. In accordance with the task of improving quality in a multi-level control system, it is necessary not only to decide what is more effective for improving quality at each stage of control and the chain of execution of decisions made, which makes it possible to determine not only the focus of financing and its ways, but also to calculate the achieved level of quality improvement.

The purpose of the article is to build a quality management strategy with discrete response functions.

2. PREVIOUS RESEARCH

A process consisting of sequential operations and their analysis were considered in [2] (deterministic parameters), [3] (local optimization), [4] (multi-criteria optimization for solving a specific problem), [5] (workflow optimization).

The article presents quality assessment in a multi-level control system with response functions of various types, optimization problems and their algorithmic solutions.

3. OPERATIONS AND ACTIVITIES

Recall the original statement from [1]. Considering the operations to be independent in aggregate and using the probability multiplication theorem [6], we obtain that the probability of a qualitative completion of the entire sequence of operations is a production:

$$p^0 = \prod_{i=1}^{n} p_i^0 . \qquad (1)$$

The probability of high-quality performance of each operation can be increased by carrying out activities. However, the ideal value of such a criterion, equal to one, can be achieved only if all probabilities are equal to 1, which is usually not practically achievable. Let's choose the threshold value of the probability of a qualitative completion of the control process [1] and transfer the target function to the category of constraints. The events can be divided into two groups:

- events with a conditionally linear response;
- events with an asymptotic response.

4. QUALITY ASSESSMENT IN A MULTI-LEVEL CONTROL SYSTEM WITH DISCRETE RESPONSE FUNCTIONS

We will carry out the formulation of the optimization problem [1] for discrete response functions. Let there be several stages (operations) S_1, S_2,..., S_n. For each operation, we know the value of the probability with which we get a qualitative completion of the i-th operation, p_i^0, i=1…N.

Let there be a set n_i of events for each i-th operation $M_{i_1}, M_{i_2}, ...M_{i_{n_i}}$, the conduct of each of which increases the probability p_i^0 by some value Δp_{ij}, i=1…N, j=1…n_i. Consider the case when one event affects only one operation and does not depend on p_i^0 (if we adhere to the terminology introduced in the previous section, then all events have an absolute conditionally linear response). Solving the problem can also be useful in purely technical applications [7, 8].

An increase in the current probability on Δp_{ij} entails some material costs c_{ij} for the implementation of the relevant event. In this statement, we will assume that c_{ij} do not depend on the initial value p_i^0. Thus, for each i-th operation, a n_i-dimensional probability vector $\Delta p_i = (\Delta p_{i_1}, \Delta p_{i_2}, ... \Delta p_{i_{n_i}})$ and a cost vector $c_i = (c_{i_1}, c_{i_2}, ... c_{i_{n_i}})$ are given.

We introduce a vector $z_i = (z_{i_1}, z_{i_2}, ... z_{i_{n_i}})$ for the i-th operation, each component of which can take the value 0 or 1:

$$z_{ij} = \begin{cases} 1, \text{if the i-th event is held for the j-th operation} \\ 0, \text{ otherwise} \end{cases} \quad (2)$$

The task is to select such a set of measures for each operation so that their implementation provides a sufficient level p_{before} of probability of the entire process of training trainees in a multi-level management system, provided that the costs of carrying out this set of measures are minimal:

$$\sum_{i=1}^{N} \sum_{j=1}^{n_i} z_{ij} c_{ij} \to \min \quad (3)$$

$$\prod_{i=1}^{N} \left(p_i^0 + \sum_{j=1}^{n_i} z_{ij} \Delta p_{ij} \right) \geq p_{before} \quad (4)$$

$$z_{ij} \geq 0, j=1 \ldots n_i, i=1 \ldots N \quad (5)$$

Let's investigate the problem (3)-(5).

Definition. Suppose there are many activities $M=\{M_i\}$. Let's call any subset M_k^* of a set M consisting of k elements a generalized event of the k-th order.

The solution of this problem by a complete search method with a sufficiently large number of activities and design stages becomes difficult due to the large number of options.

Let there be L_i activities for each design stage. Then the number of different combinations of these events of 0, 1, 2, ..., L_i pieces will be equal to $\prod_i^N 2^{L_i} = 2^{\sum_{i=1}^{N} L_i}$, where N is the number of stages.

Due to the above, it is necessary to develop an algorithm for solving the problem, which allows reducing the number of iteration options.

Since the process is divided into stages, it seems natural to consider the stages sequentially. Due to the non-linearity of constraint (4) and the fact that the multipliers included in it are modulo less than or equal to 1, the sequential selection of an event with a minimum cost at each step will not lead to an optimal solution.

Due to the above, the following algorithm for solving the optimization problem (3)-(5) is proposed.

Step 1. For each stage S_i, form a sequence of all possible generalized measures of the orders 0, 1, 2, ..., L_i. Denote the received events M^*_{ik}, k=0, 1, …, L_i; i=1, 2, …, N.

Step 2. For each generalized event M^*_{ik}, calculate the probability increment $\Delta p_{ik} = \sum_{M_j \in M^*_{ik}} \Delta p_{ij}$ and cost $c_{ik} = \sum_{M_j \in M^*_{ik}} c_{ij}$.

Step 3. Get the probabilities of qualitative completion of the stages when performing generalized activities $p_{ik} = p_i^0 + \Delta p_{ik}$. If, when summing, we get a result exceeding 1, then p_{ik} is assumed to be equal to 1.

Step 4. Select the 1st stage. Exclude from consideration those generalized measures for which $p_{ik}<p_{before}$. For the remaining generalized measures M^*_{1k}, recalculate p_{before} according to the formula $p^{before}_{1k} = \frac{p_{before}}{p_{1k}}$.

Step 5. Determine the smallest p^{min}_{1k} and largest p^{max}_{1k} probability p_{1k} values. Select a step τ and divide the segment [p^{min}_{1k}, p^{max}_{1k}] into several parts with a step τ.

For each segment of the partition, determine those generalized events whose probabilities fall into this segment. For each segment, select a generalized event with a minimum cost. Display the selected event as a node in the decision tree. Form the next level of the decision tree (second stage, i=2).

Step 6. Select the nodes $M^*_{i-1,\square}$ of the previous level sequentially. Exclude from consideration those activities (nodes) $M^*_{i-1,\square}$ that have $p_{ik} < p^{before}_{i-1,k}$.

Step 7. Determine the smallest p^{min}_{ik} and largest p^{max}_{ik} probability p_{ik} values. Divide the segment [p^{min}_{ik}, p^{max}_{ik}] into several parts in increments τ.

For each segment of the partition, determine those generalized events whose probabilities fall into this segment, and find among them a generalized event with a minimum cost. Display it as a node on the decision tree.

Step 8. Go to the next stage (increase i by 1). Repeat Steps 6 and 7 until all stages are exhausted.

Step 9. Determine the node of the last nth level having the minimum cost. Sequentially climbing the tree from children to parents, we get the values of generalized measures for all stages.

Remark. The step τ of the partition can be taken arbitrarily small and depends on the accuracy of the probability calculations. If the step value τ does not exceed the accuracy of calculations, then the optimal solution will be obtained as a result of the algorithm.

5. OPTIMIZATION TASK OF OPERATIONAL QUALITY MANAGEMENT IN A MULTI-STAGE MANAGEMENT SYSTEM

Let's now consider the case when one event can have an impact on the quality level of one or more stages at the same time, which is usually the case in practice. Consider the matrix of relative costs $\{S_{ln}\}$, each component of which is the cost of the event l (l=1..L) to increase p_n by 0.01 (can be replaced by any other sufficiently small value, Figure 1).

	1			n			N
1							
l				S_{ln}			
L							

Figure 1. The event l affects not only the probability p_n, but also the other probabilities p_i

Consider the mechanism of changing p_n. For simplicity of consideration, we assume that p_n increases by 0.01 due to the use of those measures l for which $S_{ln}\neq 0$. Several measures applied simultaneously can cumulatively change the same pn. Thus, if $z_l\neq 0$ and $S_{ln}\neq 0$, then p_n will be increased by 0.01. The resulting probability value as a consequence of the total increase as a result of the application of a set of measures can be defined as

$$p_n^* = p_n + 0.01 \cdot \sum_{l=1}^{L} z_l \delta(S_{ln}) \qquad (6)$$

$$\delta(x) = \begin{cases} 0, x \le 0 \\ 1, x > 0 \end{cases} \qquad (7)$$

Thus, condition (4) will go to

$$\prod_{n=1}^{N} \left[p_n + 0.01 \cdot \sum_{l=1}^{L} z_l \delta(S_{ln}) \right] \ge p_\Delta \qquad (8)$$

and it turns out that the variables used for optimization are included in the limitation - the task becomes significantly nonlinear.

Let us now consider expression (3) under the assumption that S_{ln} depend on the point p_n at which they are defined.

Response functions for several operations (p_i, c_l) are defined for the event l. We introduce the parameter f, which has the meaning of the initial value of the probability of the operation to which the event l is applied. Then we can get a parametric function $\Delta p_i(c_l, f)$ - the change in the probability of a qualitative completion of the operation depending on the invested funds c_l and the initial value of the probability f. Having found the inverse function $c_l(\Delta p_i, f)$, we get the value of the cost, which gives an increase in the initial probability f by Δp_i. Then, for each Δp_i=0.01 value of the parameter f from 0 to 0.99, we have the value of the cost of the event l, at which the probability of a qualitative completion of the i-th operation increases from f to f+0.01.

Now the relative cost matrix, each component of which is the cost of the event l (l=1..L) to increase p_n from f_k to f_{k+1}, is a set of 100 matrices $\{S_{ln}(f_k)\}k$, where f_k=0.01(k-1), k=1..100, or simply $\{s_{kln}\}$.

Let's analyze the change in the formulation of the optimization problem. Let the initial probabilities be p_n^0, achieved as a result of solving the problem - p_n. In this situation, we can no longer choose exactly S_{ln}, because, firstly, they will depend on the initial values p_n^0, and, secondly, they will affect p_n together.

Let's try to estimate the total cost of the events. To do this , consider a separate event l. It generally affects several probabilities p_n. Let us assume that as a result of applying a set of measures, we from p_n^0 have obtained p_n, i.e. k_{no}*0.01→ k_n*0.01.

If $k_{no}<k_n$, then it was spent on carrying out the corresponding event $\sum_{k=k_{no}}^{k_n} S_{ln}^k$.

Summing up all the stages, we get the total costs of the event l

$$S_l = \sum_{n=1}^{N} \theta_n \left\{ \sum_{k=k_{no}}^{k_n} S_{ln}^k \right\}, \tag{9}$$

where $\theta_n = \begin{cases} 1, k_n > k_{no} \\ 0, \text{otherwise} \end{cases}$.

Now we can estimate the upper limit of the costs of carrying out the entire complex of measures $\hat{S}$:

$$\hat{S} = \sum_{l=1}^{L} S_l = \sum_{l=1}^{L} z_l \left[\sum_{n=1}^{N} \theta_n \left\{ \sum_{k=k_{no}}^{k_n} S_{ln}^k \right\} \right] \tag{10}$$

Minimizing $\hat{S}$ by p_n and z_l, we get a plan for quality improvement measures. The final version of the optimization task of operational quality management:

$$\min \sum_{l=1}^{L} z_l \left[\sum_{n=1}^{N} \theta_n \left\{ \sum_{k=k_{no}}^{k_n} S_{ln}^k \right\} \right] \tag{11}$$

under restrictions

$$z_l \in \{0,1\} \tag{12}$$

$$\prod_{n=1}^{N} p_n \geq p_{before} \tag{13}$$

where $\theta_n = \begin{cases} 1, k_n > k_{no} \\ 0, \text{otherwise} \end{cases}$;

k_n = Entier(100p_n);

p_n^0 - the initial value of the probability of the qualitative implementation of stage n of management (before the implementation of the set of measures);

p_n - the final (expected) value of the probability of the qualitative implementation of stage n of management after the implementation of the set of measures.

The exact solution of the problem (11)-(13) is not possible due to its NP-completeness, in further research an appropriate numerical algorithm will be developed.

7. CONCLUSION

1. The proposed concept of probabilistic quality management in a multi-level control system is based on the assumption that there is some stable level (probability) of obtaining correct control results and differs in ways of influencing this probability.

2. The use of the concept of probabilistic quality management in a multi-level management system ensures the direction of the impacts of the management system to ensure maximum effect with fixed resources.

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